Efficient Construction of Hierarchical Overlap Graphs

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Outline

- Introduction
- Preliminaries
- Main algorithm
- Improvement using segment tree

Introduction

- Genome sequencing
  - DNA assembly: Obtaining a whole genome sequence from sequencing reads
  - Seeking some path in a graph that encodes suffix-prefix overlaps

- Overlap encoding graphs
  - Overlap graph
  - De Bruijn graph

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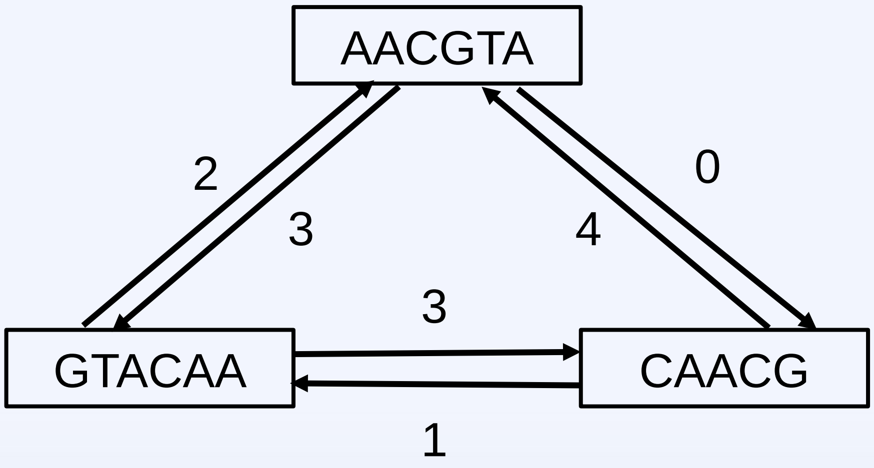

Shortest Superstring problem

- Shortest Superstring problem
  - Let P be a set of input strings.
  - Find the shortest string that contains all input strings as substring.

- Equivalent to finding a maximum weighted Hamiltonian path in the overlap graph.

- Example P := { AACGTA, GTACAA, CAACG }

```
CAACGTACAA
CAACG
 AACGTA
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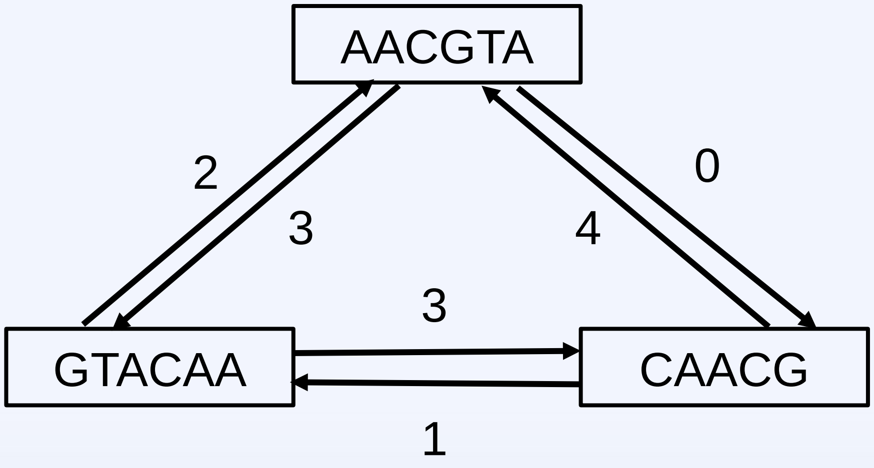

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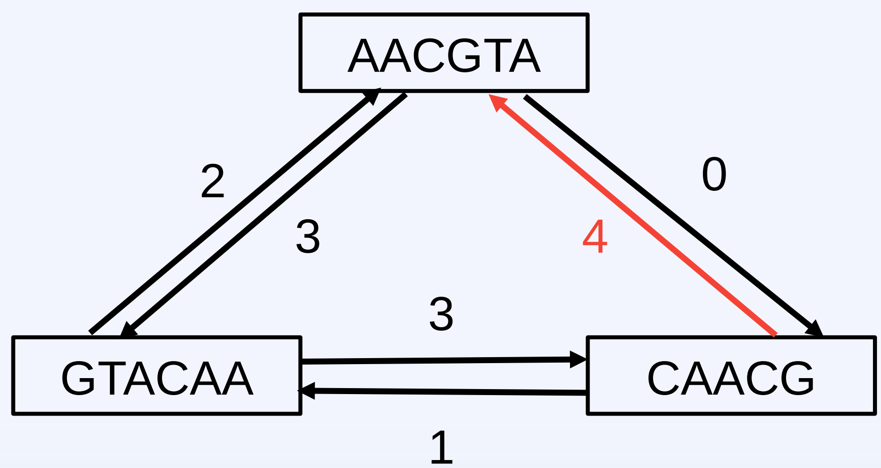

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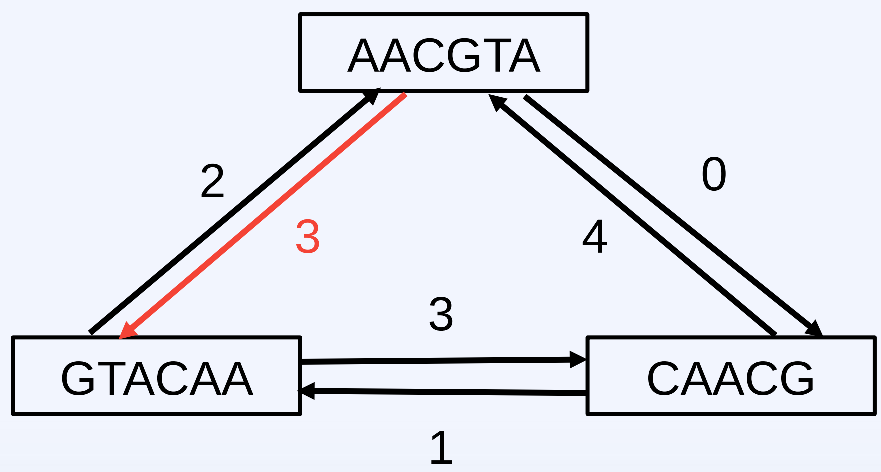
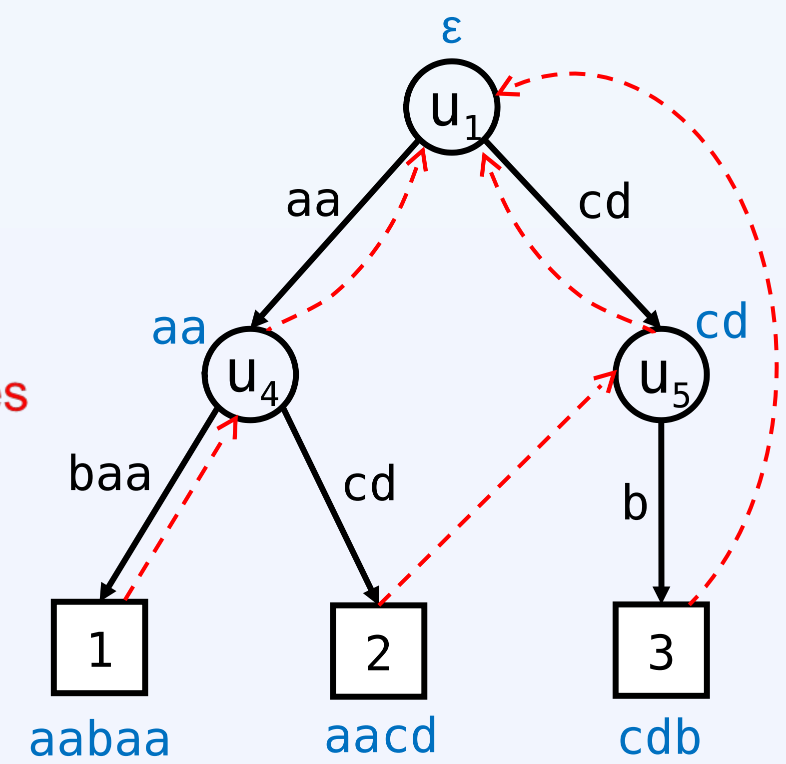


Hierarchical Overlap Graph (HOG)

- Hierarchical overlap graph (HOG)
  - First proposed by Cazaux and Rivals
  - Given a set of strings,
    - Nodes: Maximal overlaps between strings
    - Arcs: Prefix or suffix relations between nodes
  - We can divide arcs into tree edges (prefix relations) and failure links (suffix relations).



- Example
  - $S = \{aabaa, aacd, cdb\}$

Hierarchical Overlap Graph (HOG)

- Extended HOG (EHOG)
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  - Given a set of strings,
    - Nodes: (Possibly not maximal) Overlaps between strings
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- Example
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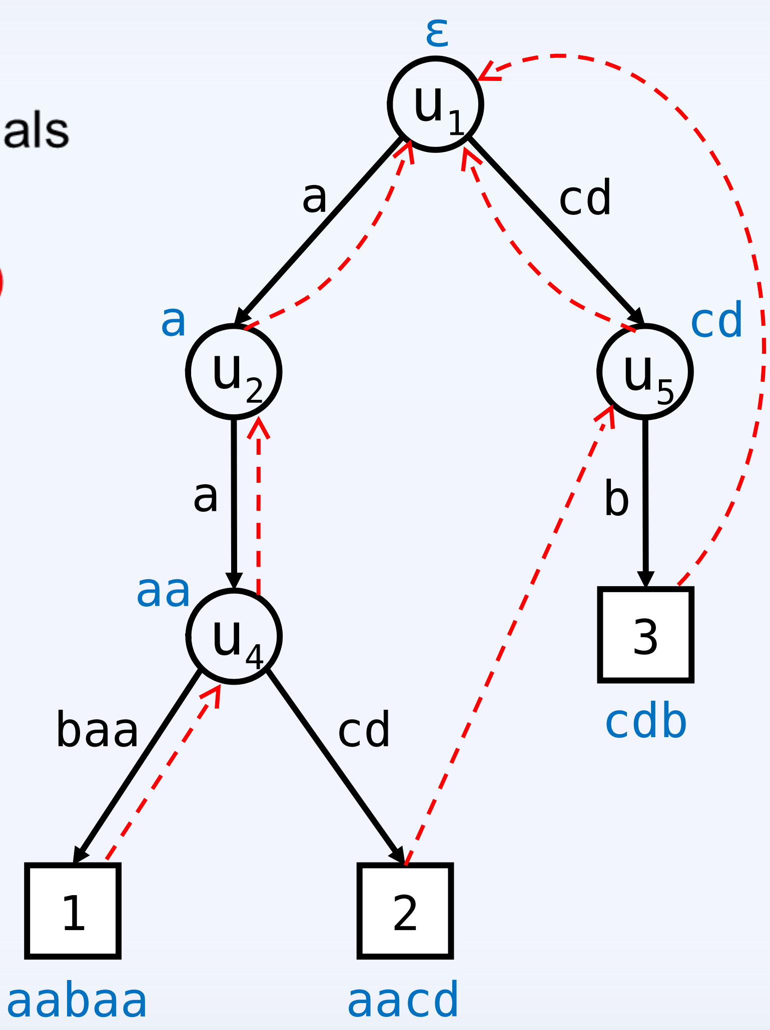
- EHOG may use considerably more space than HOG.



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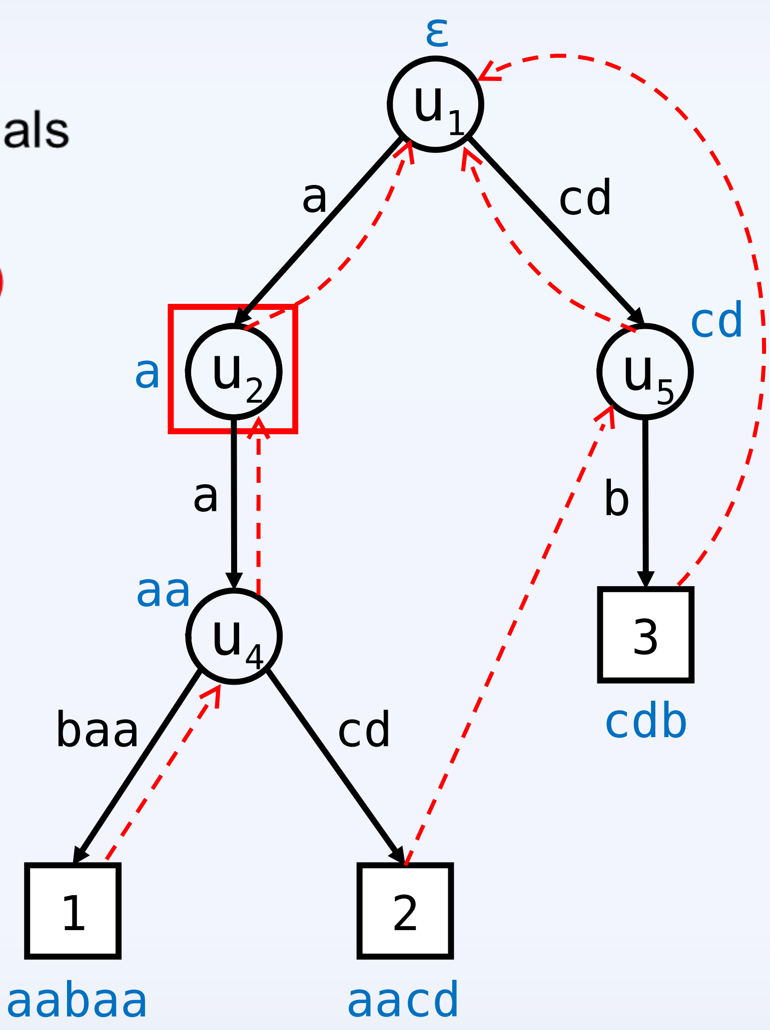


Hierarchical Overlap Graph (HOG)

- There are many advantages of HOG.

  ||P|| denotes the sum of lengths of strings in P

- HOG uses less space than overlap graph.
  - Overlap graph: $O(||P|| + n^2)$
  - HOG: $O(||P||)$

- HOG has more information than the overlap graph.
  - HOG encodes a relationship between the overlaps.

    e.g. All identical overlaps are encoded into a unique node in HOG.

- HOG has a great potential in studying the shortest superstring problem.

HOG: construction

- EHOG can be constructed in $O(||P||)$ time and space. (Cazaux B. and Rivals E., 2020)
- HOG uses more time and space.
  - Time: $O(||P|| + n^2)$
  - Space: $O(||P|| + n \times \min(n, \max\{|s| : s \in P)\}))$
- We present an algorithm using less time and space.
  - Time: $O(||P|| \log n)$ or $O(||P|| \frac{\log n}{\log \log n})$
  - Space: $O(||P||)$

Preliminaries

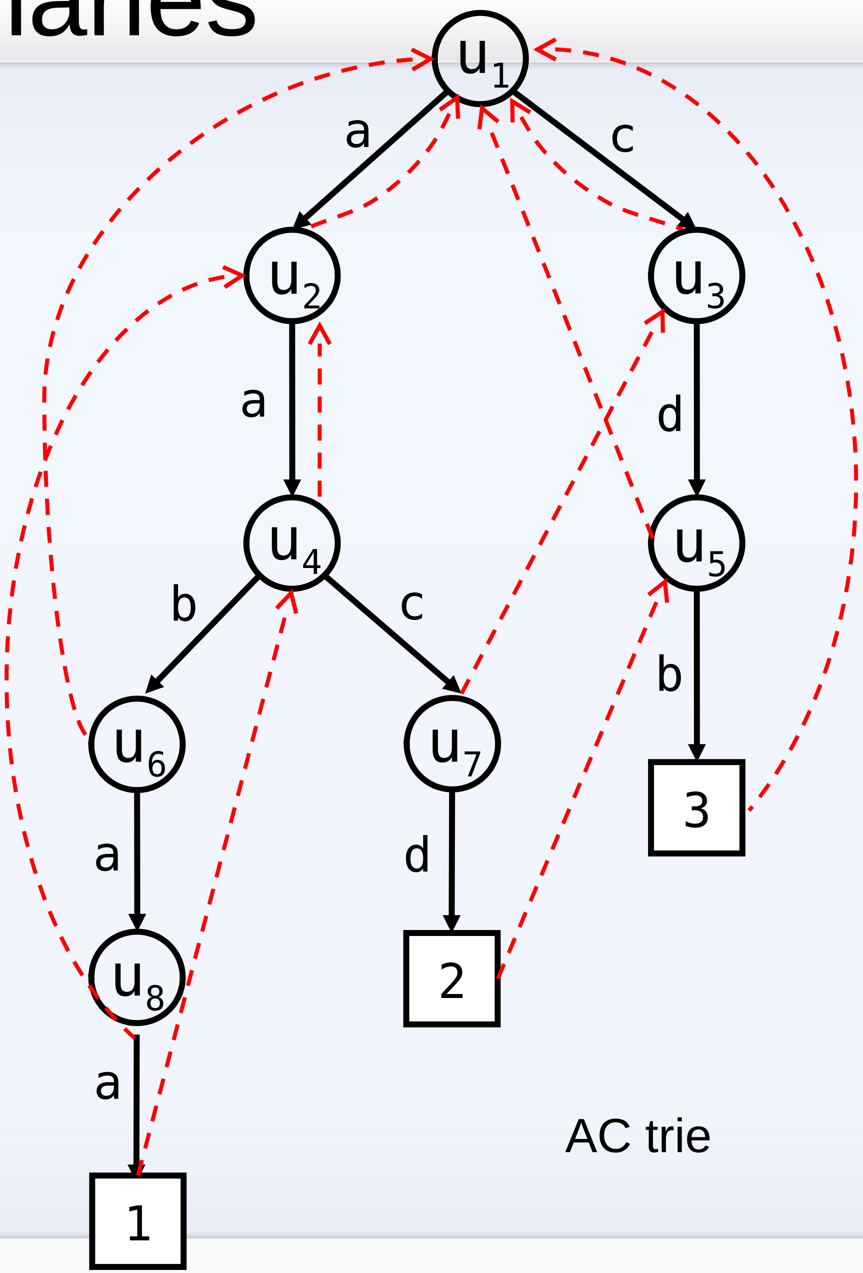
- Given a set of strings $P = \{s_1, s_2, \ldots, s_\mathrm{n}\}$,
  - $Ov^+(P)$: Set of all overlaps from s_i to s_j for $1 \leq i, j \leq n$
  - $Ov(P)$: Set of the longest overlap from s_i to s_j for $1 \leq i, j \leq n$

- In order to compute HOG, we need to compute $Ov(P)$.

- If we have $Ov(P)$ and $EHOG(P)$, we can compute $HOG(P)$ in $O(||P||)$ time. (Cazaux B. and Rivals E., 2020)

Preliminaries

- EHOG is a contracted form of an Aho-Corasick trie.
- HOG is a contracted form of an EHOG.
- Example
  - $S = \{aabaa, aacd, cdb\}$



AC trie

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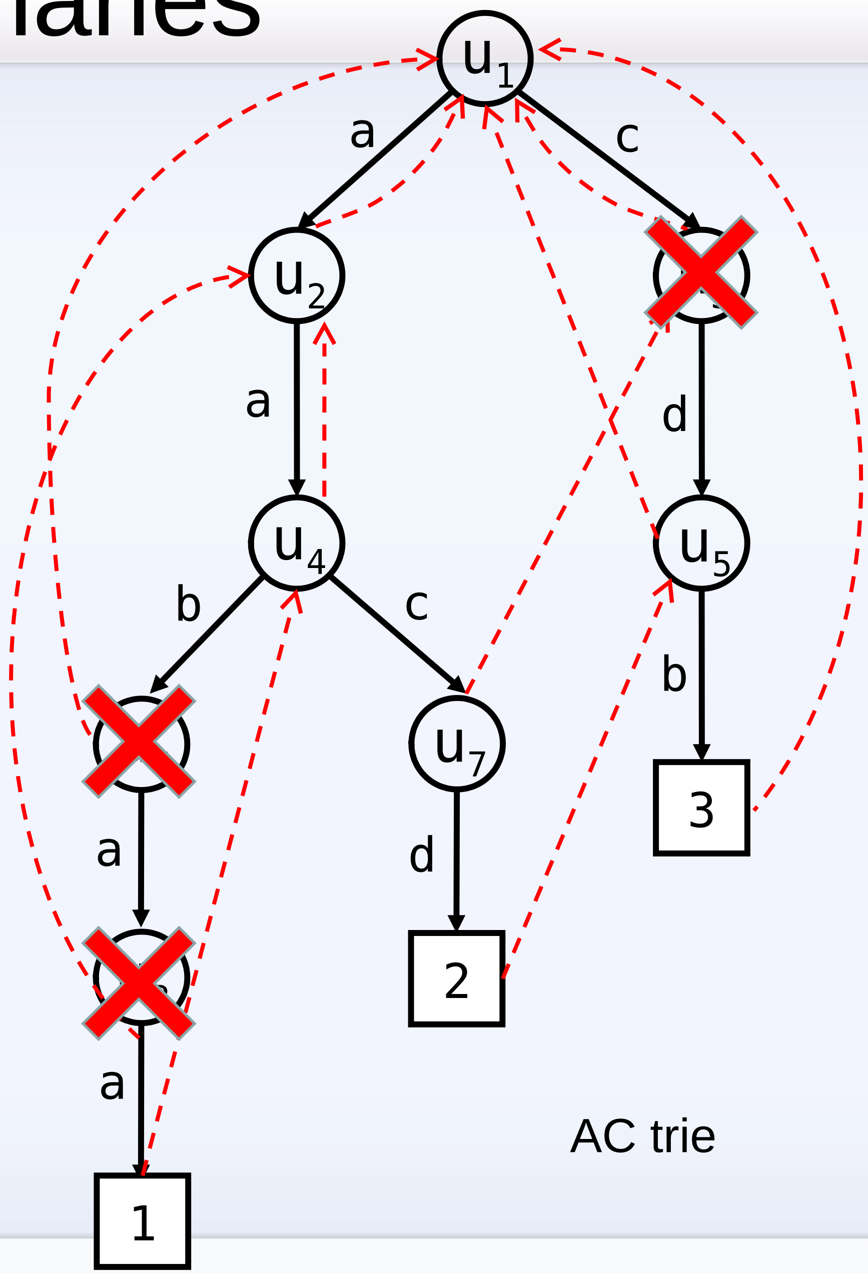

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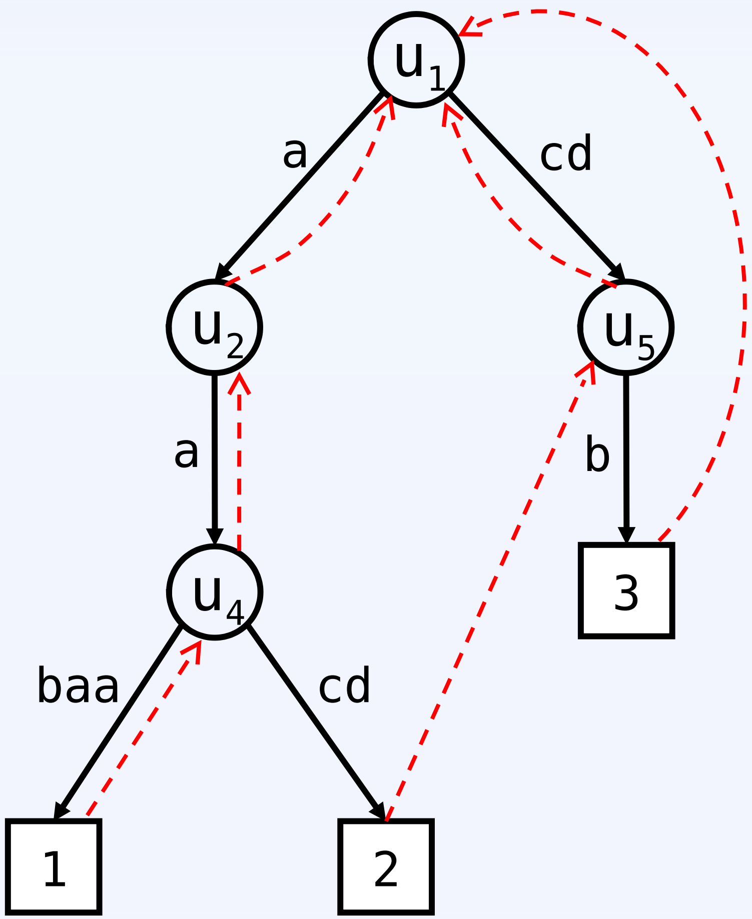

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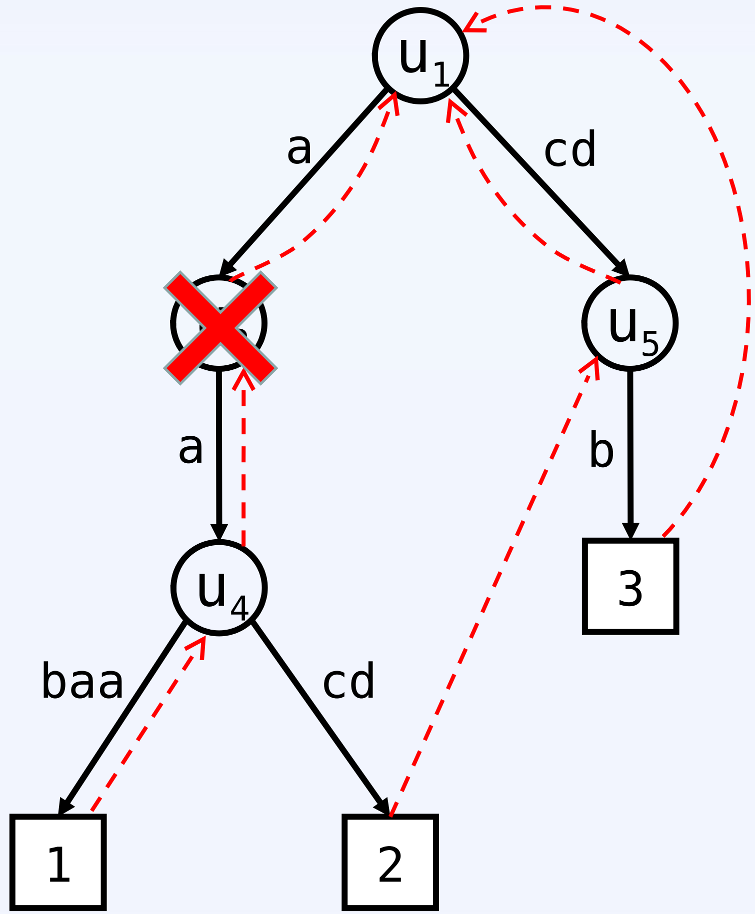

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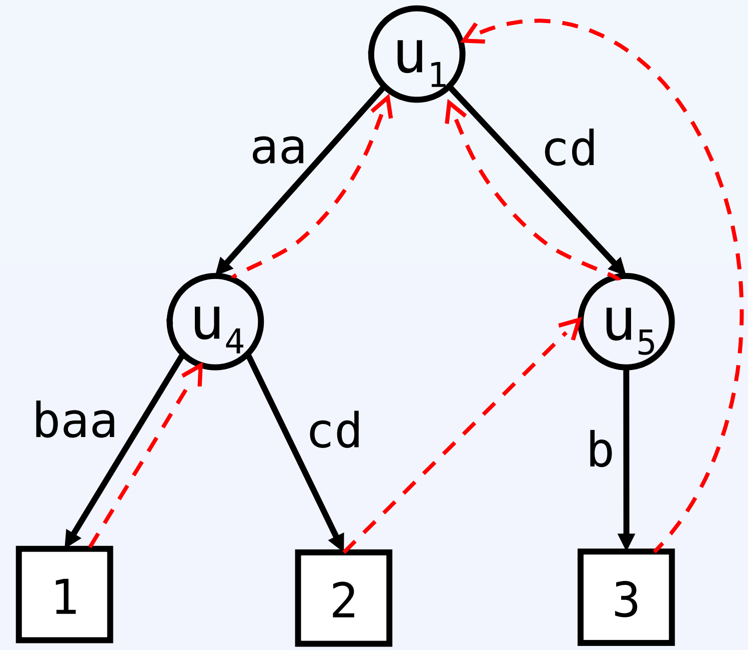


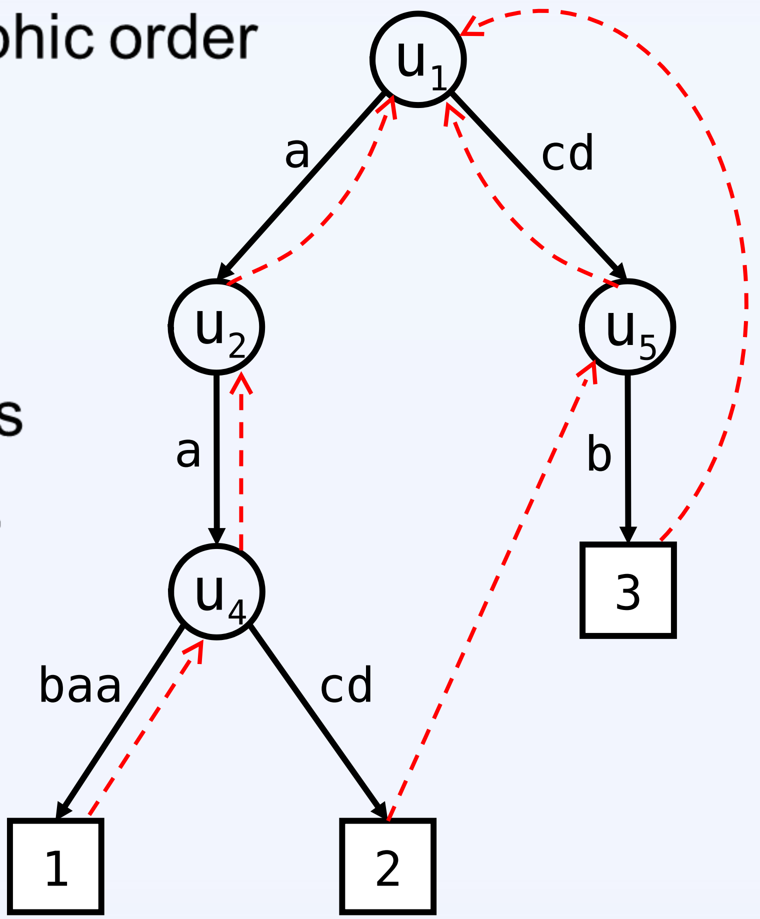
HOG

Main Algorithm

- Build an Aho-Corasick trie of P
- Renumber the strings in lexicographic order
- Build EHOG(P) in $O(||P||)$ time

- For each node u in EHOG(P), define an interval $I(u)$ that contains every leaf node in the subtree of u.

- Example: $I(u_1) = \{1, 2, 3\}$
  $I(u_2) = \{1, 2\}$
  $I(u_5) = \{3\}$



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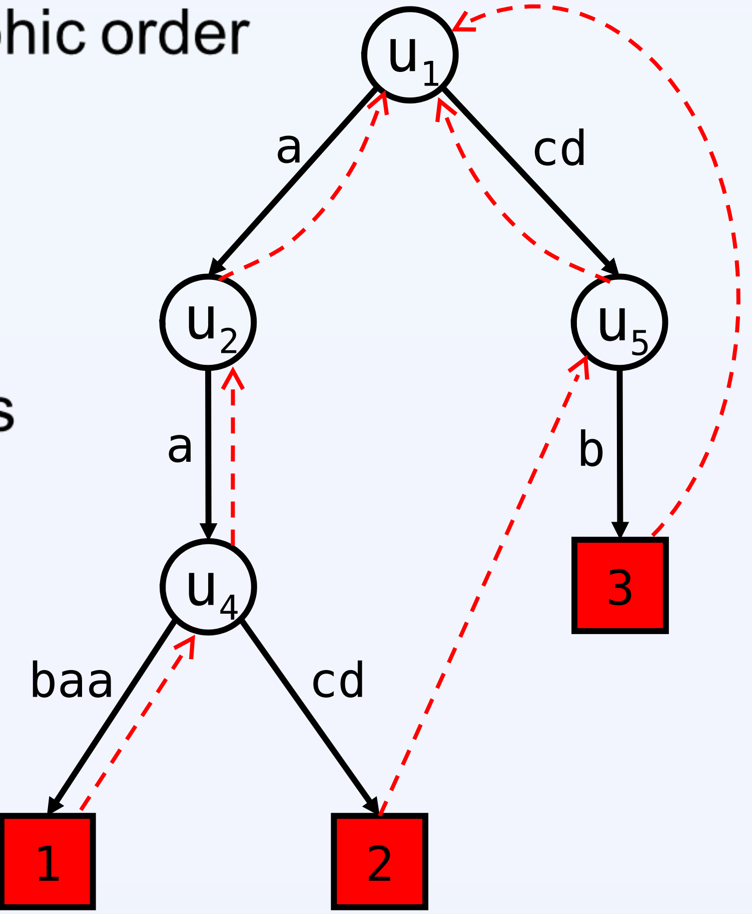

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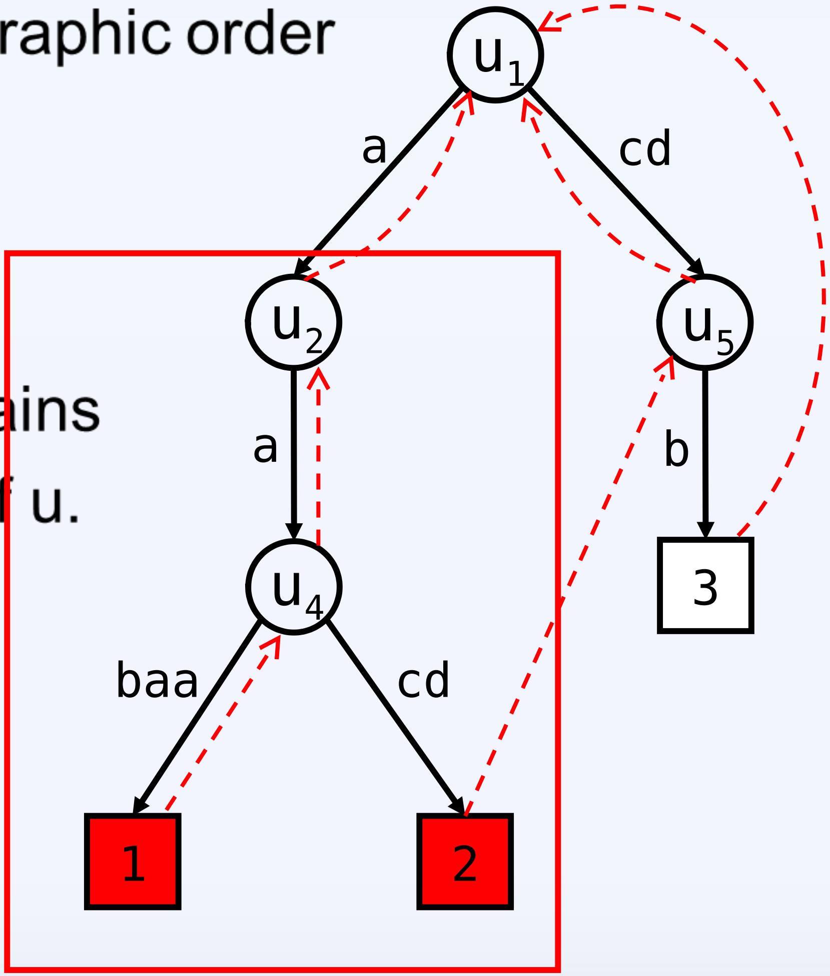

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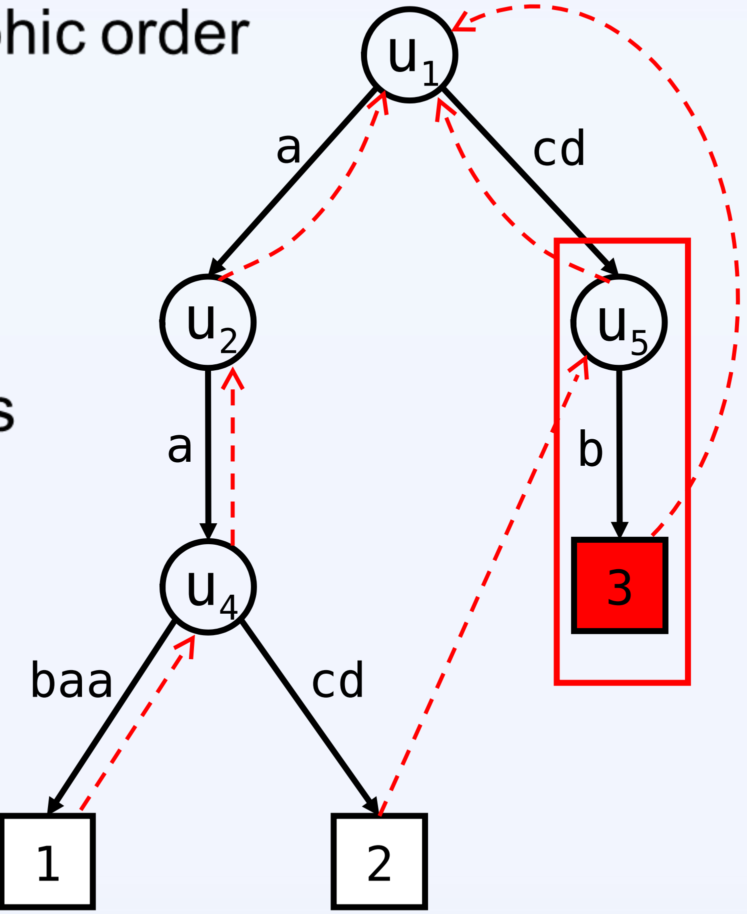


Main Algorithm

- We will compute $Ov(P)$ from $EHOG(P)$.

- What happens if $u \in Ov(P)$?
- For some $s_i, s_j \in P$, u is the longest overlap from s_i to s_j.
  - u is a proper suffix of s_i.
  - u is a proper prefix of s_j.
  - There are no longer overlaps from s_i to s_j than u.

Main Algorithm

- If we follow the failure link from s_i, we get every suffixes of s_i in a decreasing order of lengths.

- If we have a failure link chain $v_0 = s_i, v_1, v_2, \dots, v_k = root$, v_x is the longest overlap from s_i to s_j if v_x is the first node that is a prefix of s_j during the traversal.

- In other words, v_x is the longest overlap from s_i to s_j if:
  - v_x is a prefix of s_j
  - v_y is not a prefix of s_j for $1 \leq y < x$

Main Algorithm

- Maintain a bit vector B of length n.

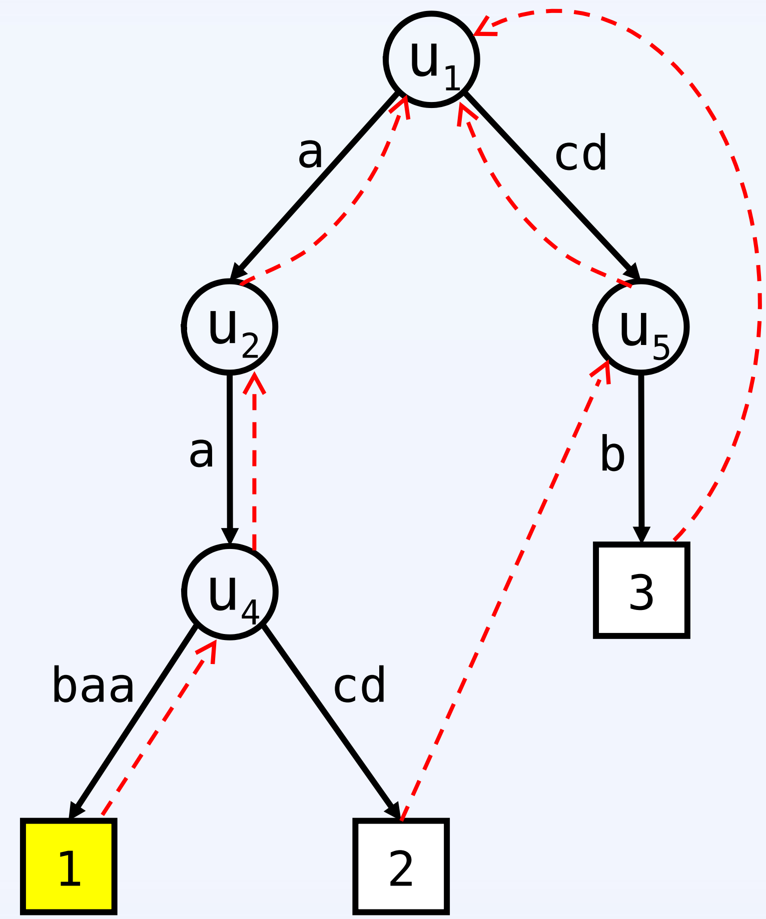
- At the end of iteration with v_x, $B[j] = true$ if and only if there exists $1 \leq y \leq x$ such that v_y is a prefix of s_j.

- We can check whether v_x should be included in $HOG(P)$ using B.

Main Algorithm

- We start with $i = 1, s_i = aabaa$.
- Initialize B with false.

- Follow the failure link repeatedly.



j	1	2	3
B[j]	false	false	false

Main Algorithm

- We start with $i = 1, s_i = aabaa$.
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- $v_1 = u_4$: $I(u_4) = \{1,2\}$

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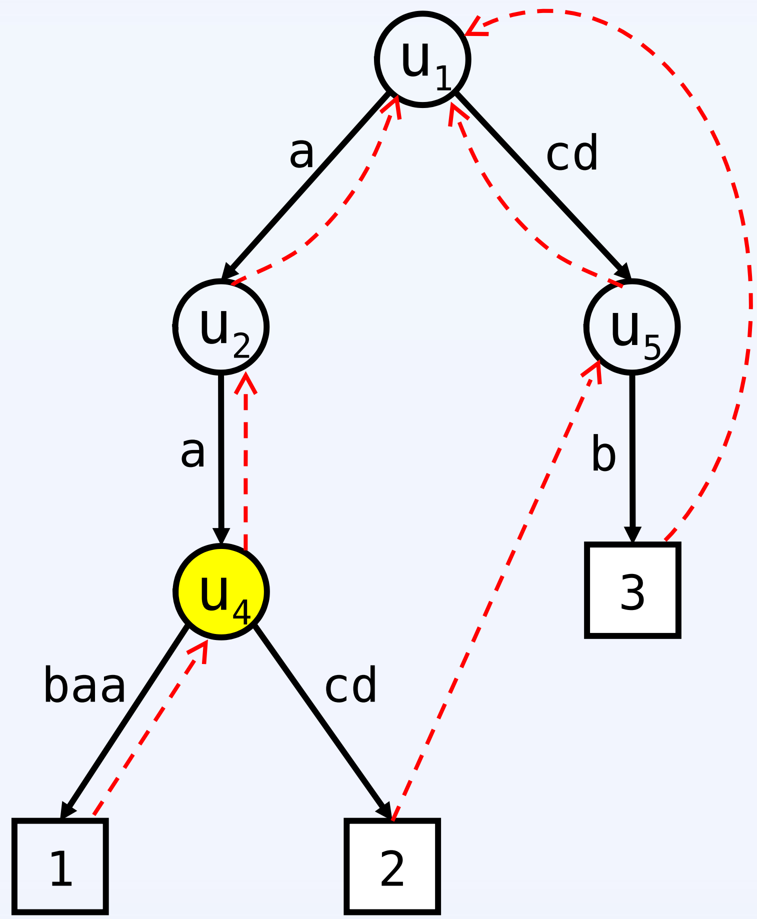

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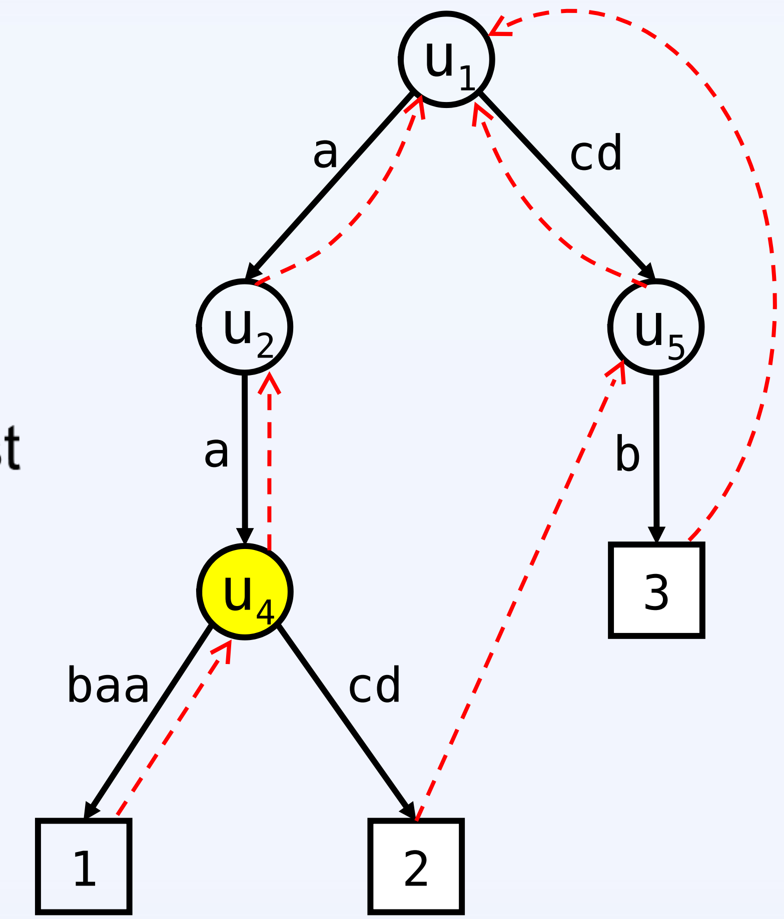
- Initialize B with false.

- Follow the failure link repeatedly.
- $v_1 = u_4$: $I(u_4) = \{1,2\}$
- B[1] = B[2] = false: u_4 is the longest overlap from s_1 to s_1 and s_2.

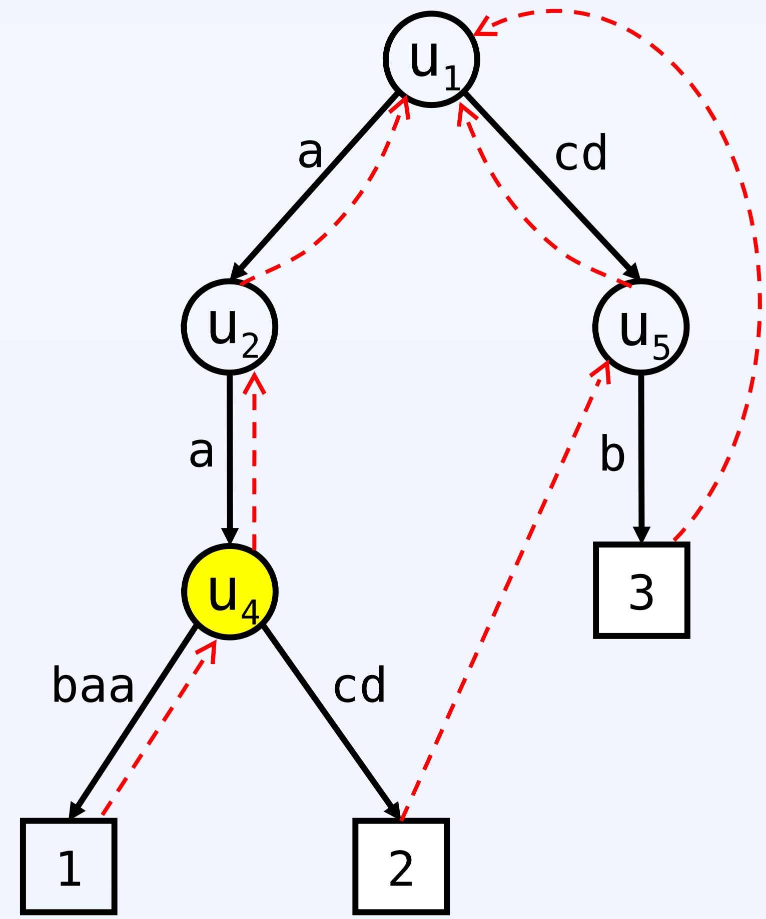
j	1	2	3
B[j]	false	false	false



Main Algorithm

- We start with $i = 1, s_i = aabaa$.
- Initialize B with false.
- Follow the failure link repeatedly.
- $v_1 = u_4$: $I(u_4) = \{1,2\}$
- Update B[1] and B[2] as true.

j	1	2	3
B[j]	true	true	false

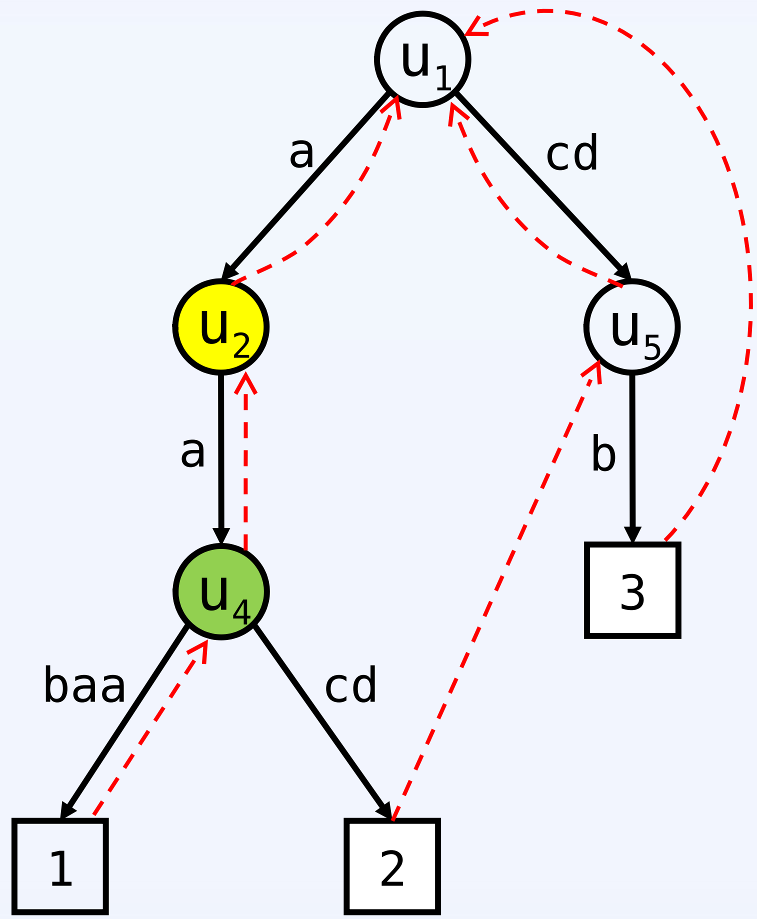


Main Algorithm

- We start with $i = 1, s_i = aabaa$.
- Initialize B with false.

- Follow the failure link repeatedly.
- $v_2 = u_2$: $I(u_2) = \{1,2\}$
- B[1] and B[2] are already true: u_2 is an overlap from s_1 to s_1 and s_2, but not the longest one.

j	1	2	3
B[j]	true	true	false

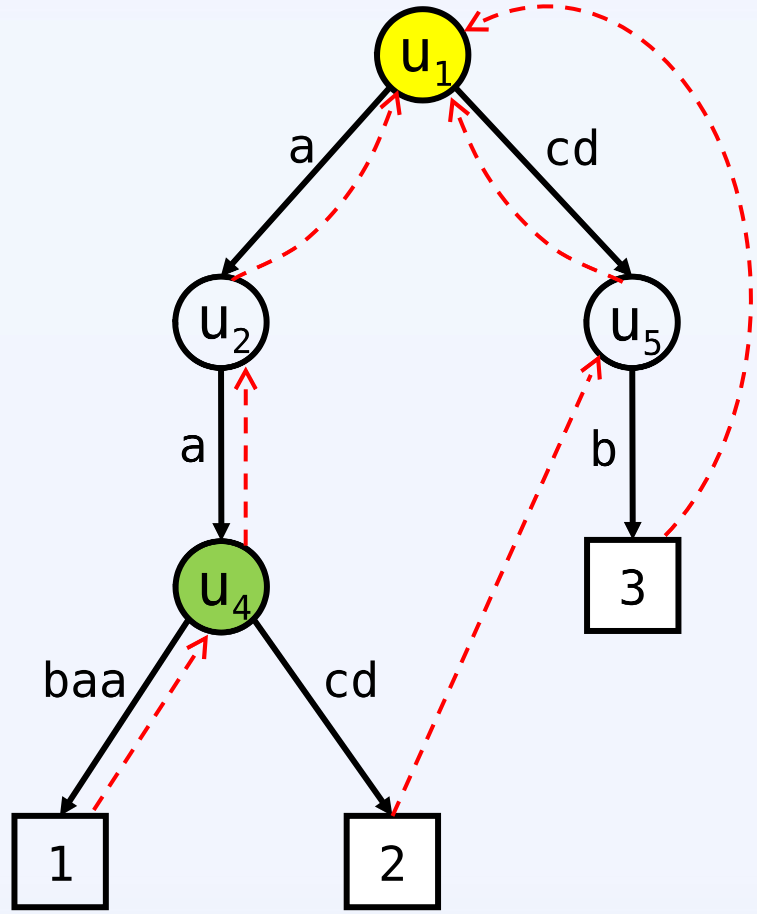


Main Algorithm

- We start with $i = 1, s_i = aabaa$.
- Initialize B with false.

- Follow the failure link repeatedly.
- $v_3 = u_1$: $I(u_1) = \{1,2,3\}$
- B[3] = false: u_1 is the longest overlap from s_1 to s_3.
- Update B[3] as true.

j	1	2	3
B[j]	true	true	true

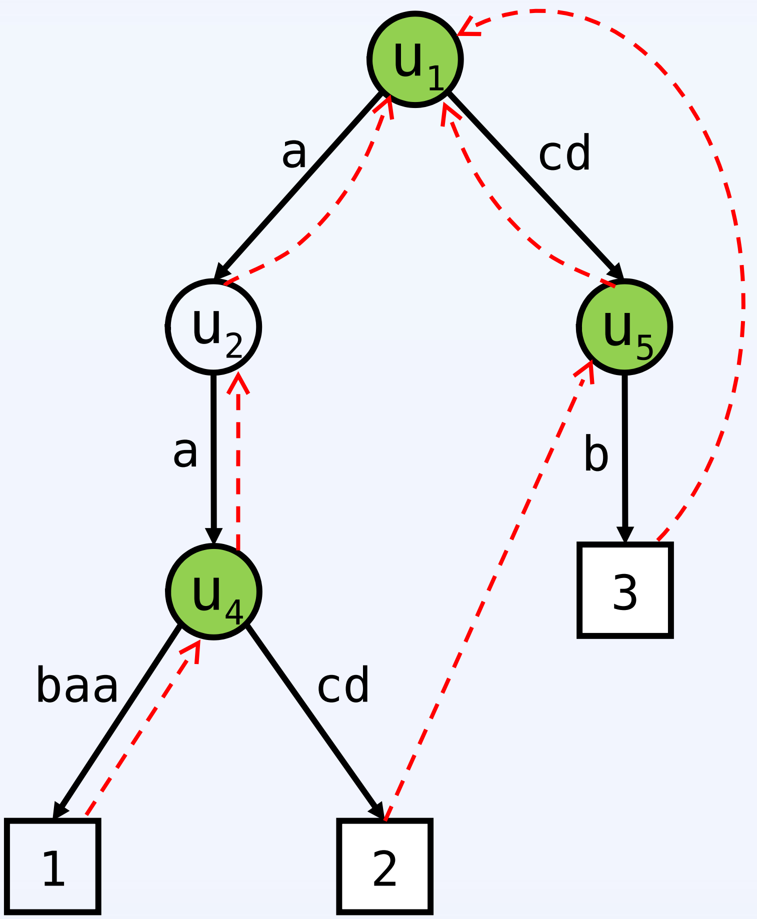


Main Algorithm

- We start with $i = 1, s_i = aabaa$.
- Initialize B with false.

- We do the same procedure starting with $s_2 = aacd$ and $s_3 = cdb$.

- $Ov(P) = \{u_1, u_4, u_5\}$



Main Algorithm (summary)

- For each s_i, do the following algorithm separately.

- Initialize B[1..n] to false
- Starting from node s_i, follow the failure links while doing the following works:

  i) If there exists $j \in I(u)$ such that B[j] = false, mark u to be included in $Ov(P)$.

  ii) For $j \in I(u)$, update B[j] as true.

- Build a HOG with marked nodes and EHOG.

Improvement using segment tree

- We need to process these two types of queries on B:

  i) If there exists $j \in I(u)$ such that B[j] = false, mark u to be included in $Ov(P)$.

  ii) For $j \in I(u)$, update B[j] as true.

- Consider an integer array A and following queries on A.

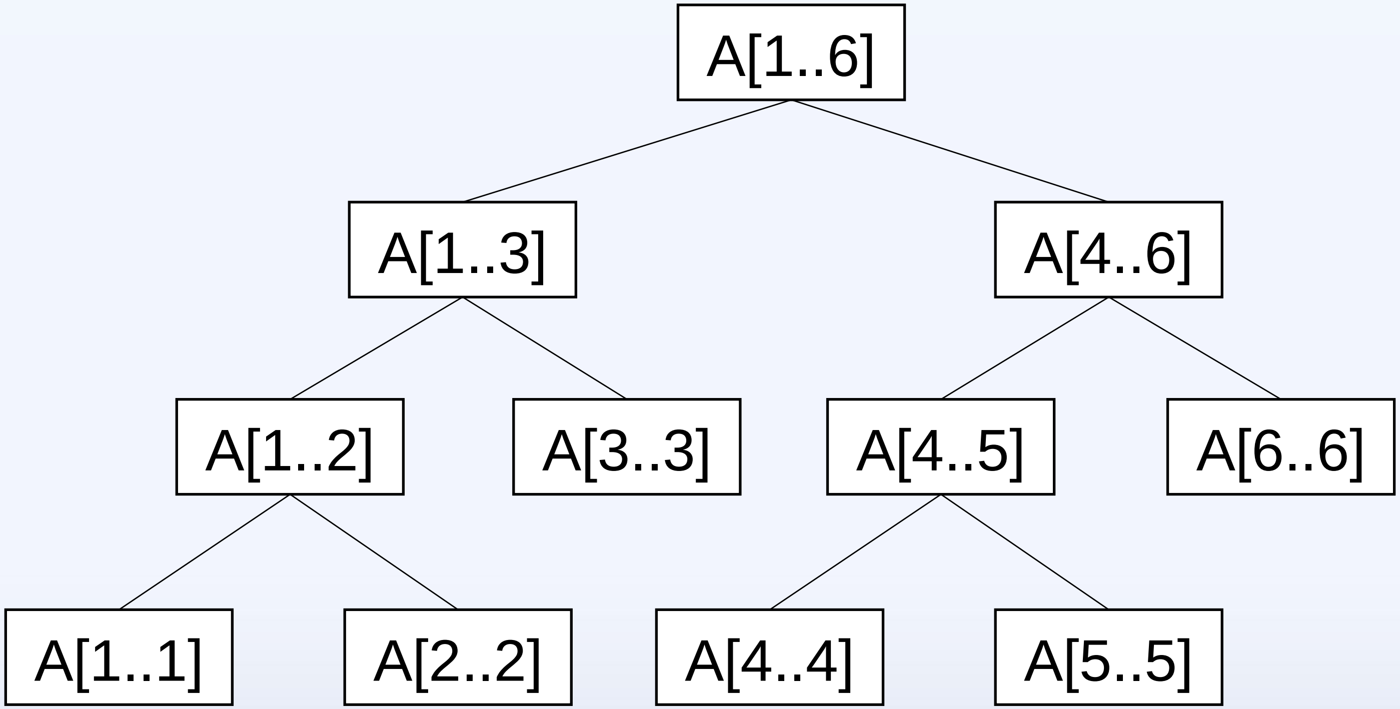
  i) Given an interval $[a..b]$, compute the minimum value among $A[a..b]$ (and check whether it is zero or not)

  ii) Given an interval $[a..b]$, add 1 to each element of $A[a..b]$.

- $A[j] = 0 \leftrightarrow B[j] = \text{false}, \text{A[j]} > 0 \leftrightarrow B[j] = \text{true}$

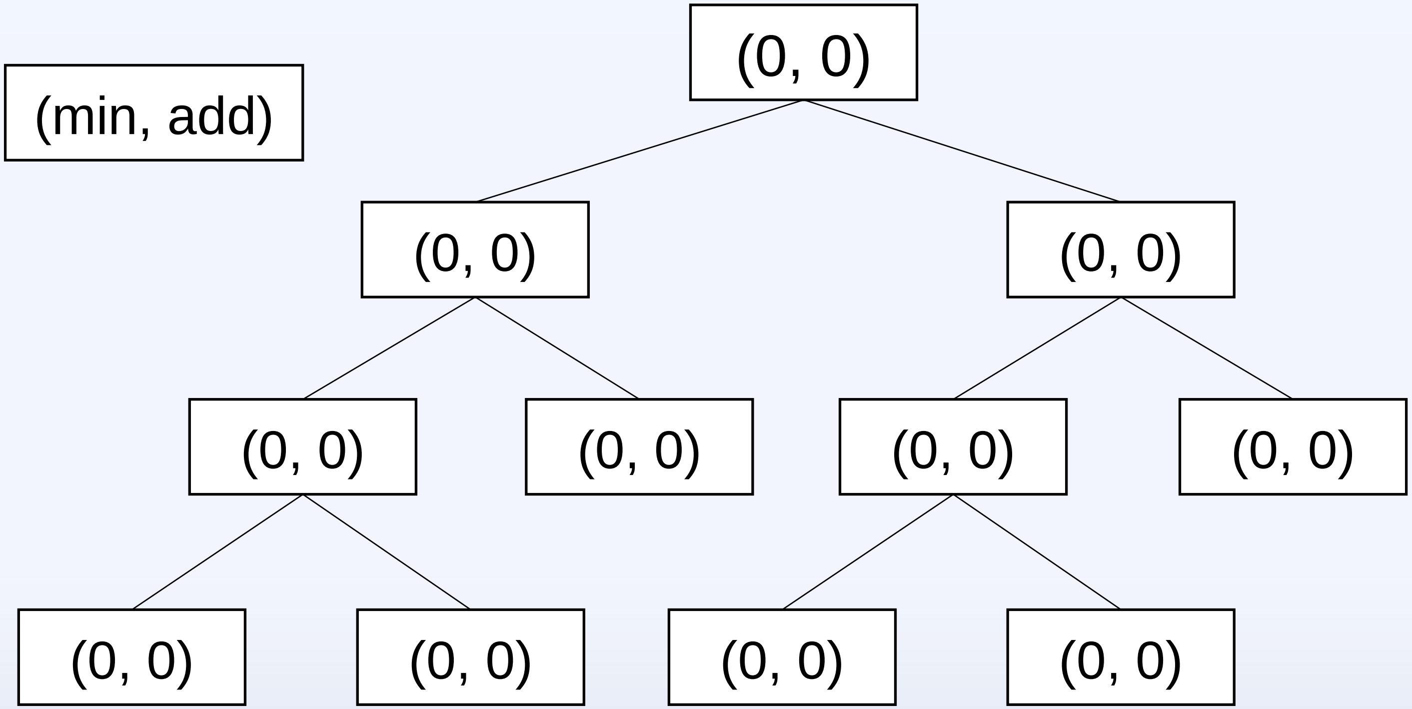
Improvement using segment tree

- We use segment tree to process queries on A.
- Segment tree: A binary tree which has n leaf nodes and has $O(\log n)$ height.
- Each internal node u corresponds to an interval $u.int$



Improvement using segment tree

- Each node stores two values, min and add.
  - $u.min$: Minimum value among the elements in $u.int$
  - $u.add$: Collectively added value to the elements in $u.int$



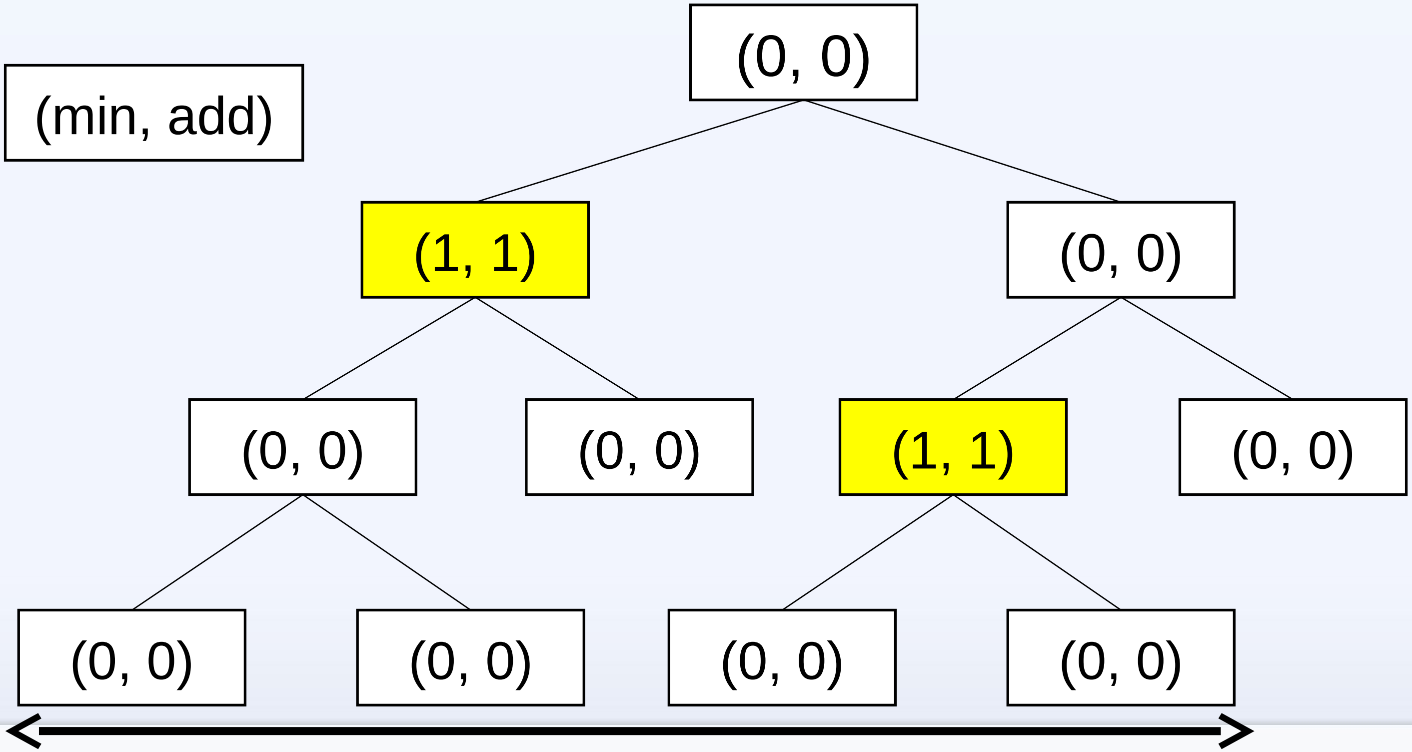
Improvement using segment tree

- We use lazy propagation technique.
- If query 1 occurs, we follow the nodes recursively from top to down, starting from the root.

- Considering a node u, we split the cases
- $u.int$ is included in the query interval: return $u.min$
- $u.int$ is disjoint with the query interval: return ∞
- Otherwise:
  - Propagate $u.add$ to child nodes
  - Continue with child nodes, and return minimum among them.

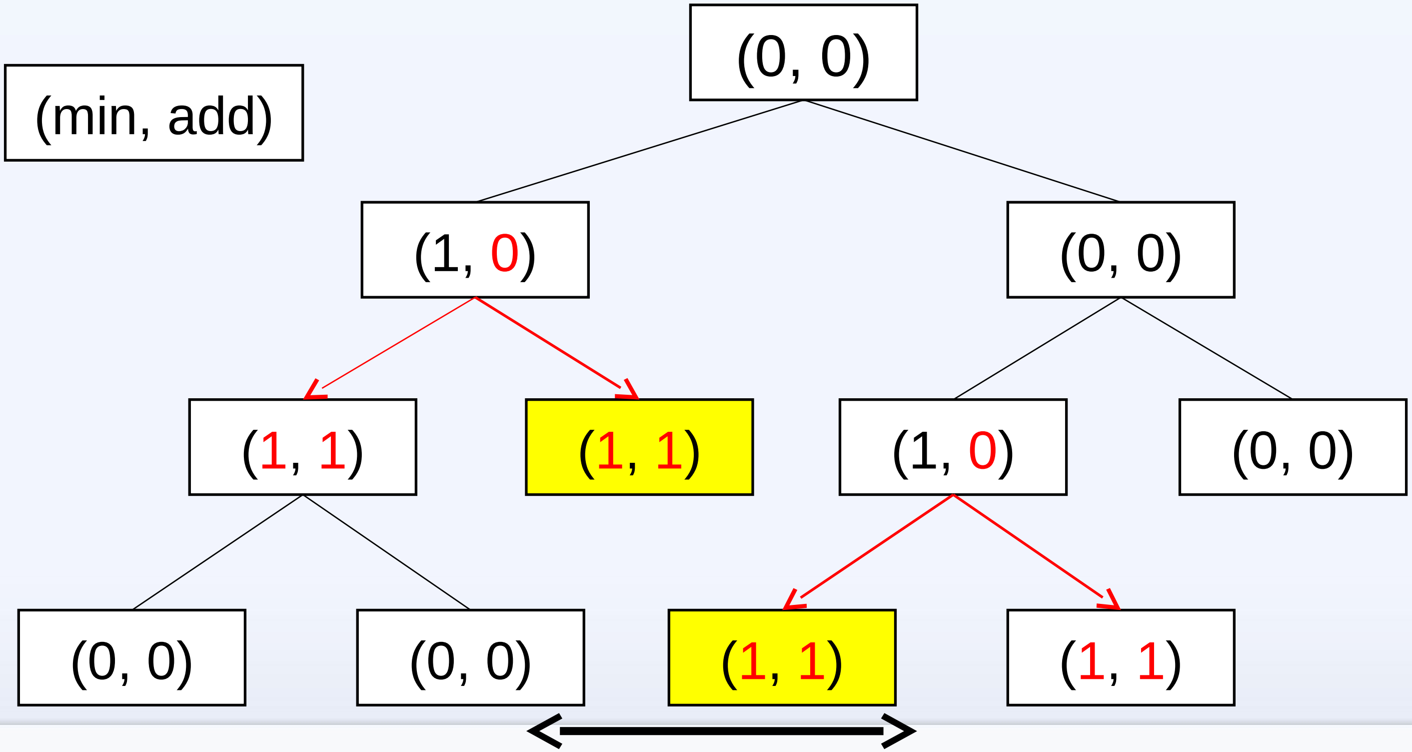
Improvement using segment tree

- Example: Query 2 on A[1..5]
  - Update two nodes, representing A[1..3] and A[4..5]
  - Their children are not updated yet



Improvement using segment tree

- Example: Query 1 on A[3..4]
  - We get min from two nodes, A[3..3] and A[4..4]
  - add value of parent nodes are propagated to their child nodes, ensuring the appropriate min values.



Improvement using segment tree

- Any interval $[a..b]$ can be represented by $O(\log n)$ nodes in the segment tree.

- Both queries can be done in $O(\log n)$ time.
- The total number of queries are $O(||P||)$.

- Time complexity: $O(||P|| \log n)$

- Both HOG and segment tree costs $O(||P||)$ space.
- Space complexity: $O(||P||)$

Conclusion

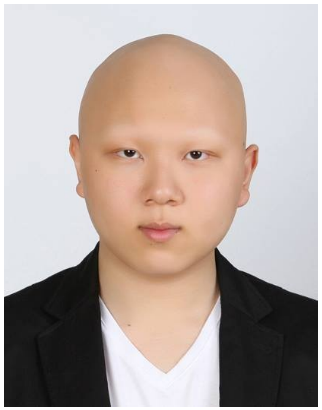
- We have presented a new algorithm to compute HOG in $O(\|P\| \log n)$ time and $O(\|P\|)$ space, using segment tree.

- We improved the time complexity of our algorithm to $O\left(\|P\| \frac{\log n}{\log \log n}\right)$, using word RAM model / w-segment tree.

Open questions :

- can one get a linear time algorithm?
- can one extend the data structure to encode approximate overlaps?

Thanks for your attention



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Improvement using word RAM

- Word RAM model: can read/write/do a bitwise operation for w-bit machine words in $O(1)$ time. $(w \geq \log n)$

- By using bitwise operations, we can improve the running time of two queries from $O(\log n)$ to $O\left(\frac{\log n}{\log \log n}\right)$.

- We use the w-segment tree, which is the w-ary version of the segment tree.

Improvement using word RAM

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Improvement using word RAM

- Instead of $u.min$ and $u.add$, we store two bit vectors of length w, $u.Vmin$ and $u.Vadd$.
- If a node u is the j-th child of its parent p, $p.Vmin[j]$ is true if and only if $u.min = 0$.
- We can update the w-segment tree similarly to the (binary) segment tree, but in $O(\log_w n) = O\left(\frac{\log n}{\log\log n}\right)$ time.
- We can simulate w-segment tree using arrays, which results in using $O(n)$ bits in total.

Pointers

Lazy Propagation in Segment Tree

https://www.geeksforgeeks.org/lazy-propagation-in-segment-tree/