Efficient enumeration of regex matches

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[2]CNRS CRIStAL

[3]CNRS CRIL

[4]Universität Bayreuth

Problem: Finding Patterns in Text

- We have a long text T:

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  - Write the pattern as a regular expression:

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→ How to find the pattern P efficiently in the text T?

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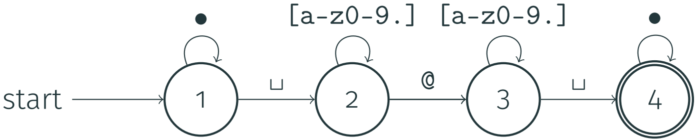

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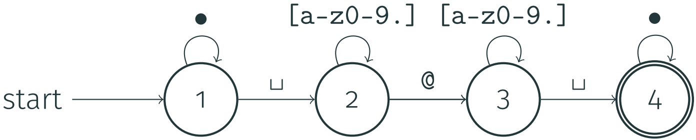

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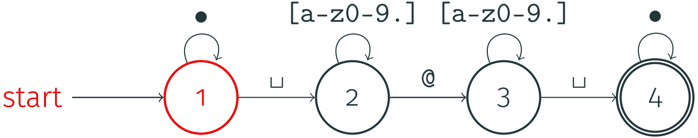

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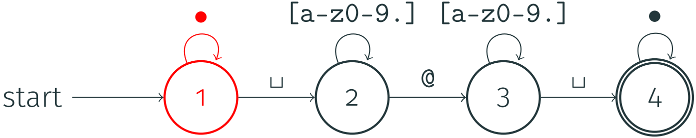

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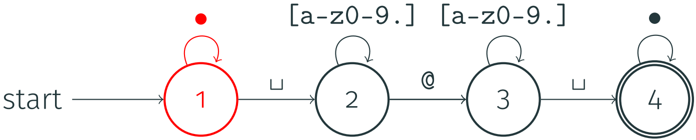

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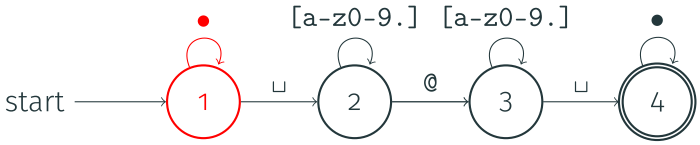

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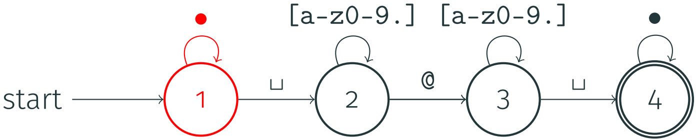

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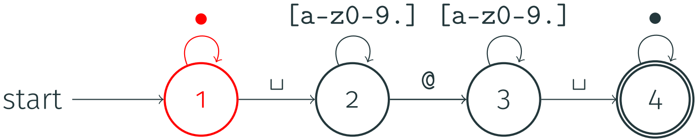

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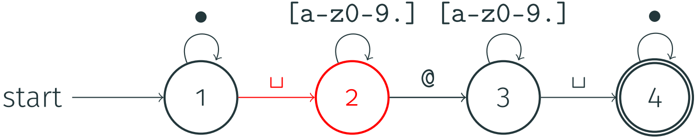

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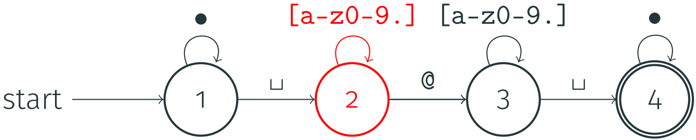

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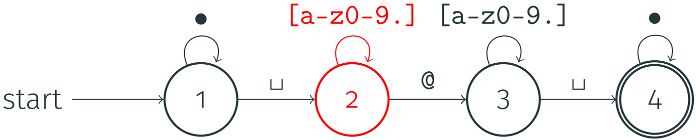

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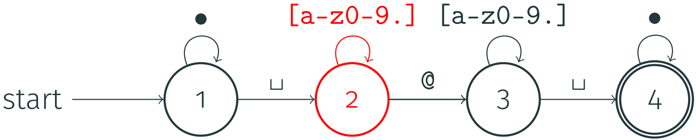

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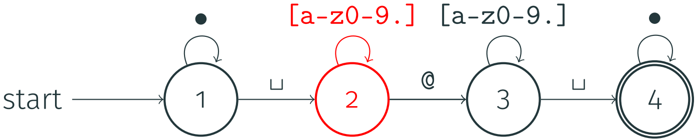

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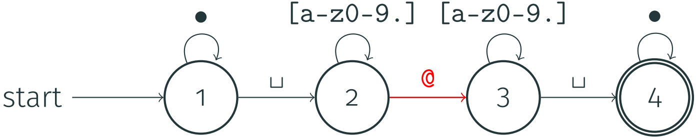

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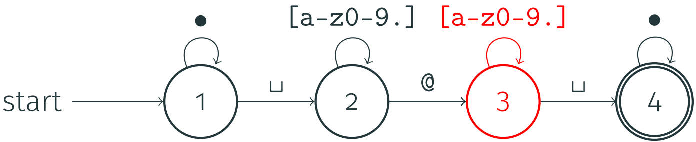

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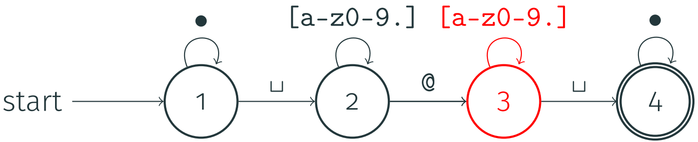

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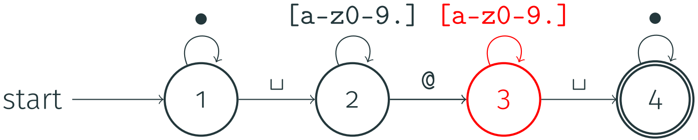

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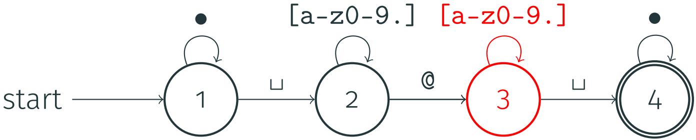

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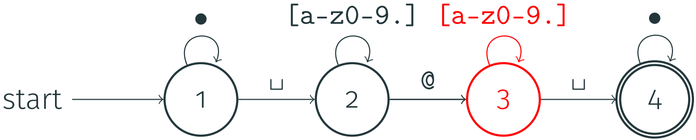

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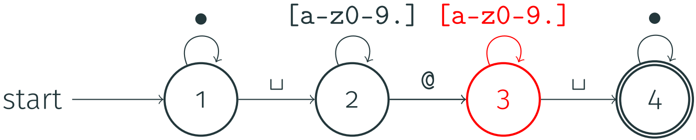

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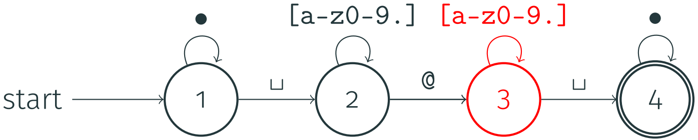

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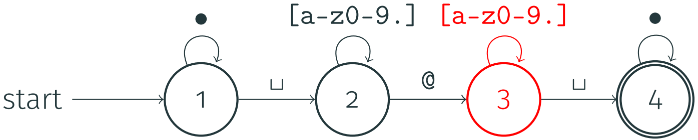

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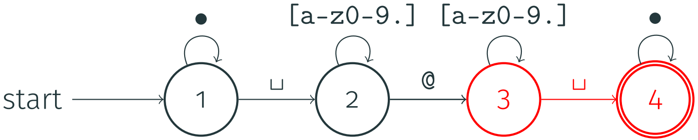

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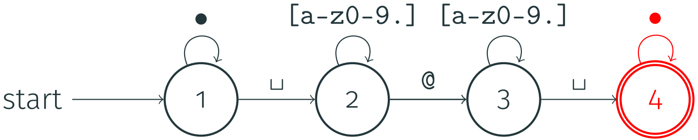

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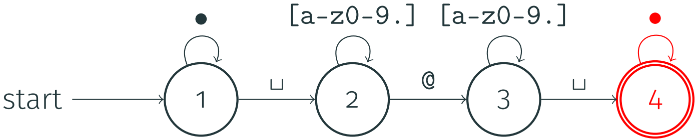

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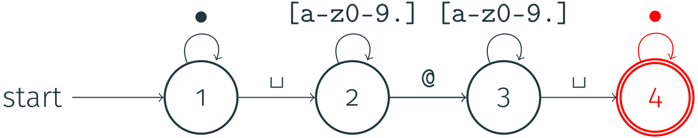

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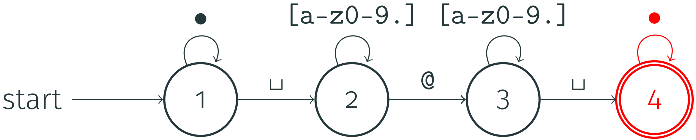

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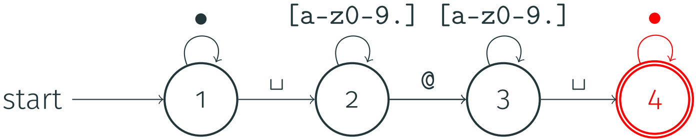

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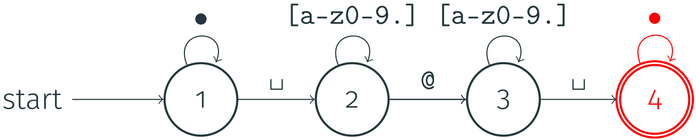

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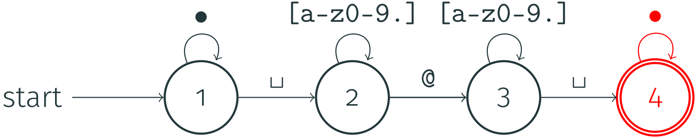

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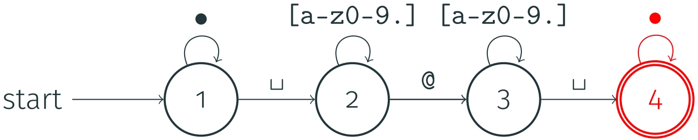

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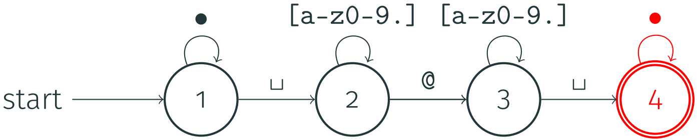

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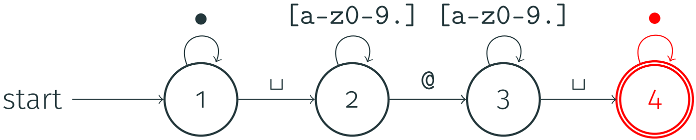

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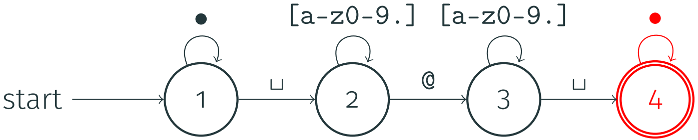

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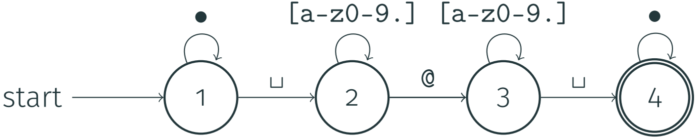

- Then, evaluate the automaton on the **text** T

`Email␣a3nm@a3nm.net␣Affiliation`

- The **complexity** is $O(|A| \times |T|)$, i.e., **linear** in T and **polynomial** in P

Solution: Automata

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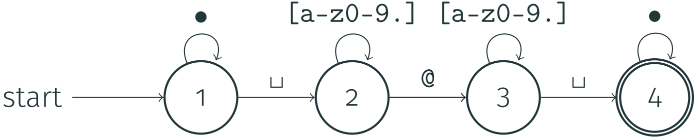

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0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
E	m	a	i	l	␣	a	3	n	m	@	a	3	n	m	.	n	e	t	␣	A	f	f	i	l	i	a	t	i	o	n

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- Naive algorithm: Run the automaton A on each substring of T

l	o	l

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$[\rangle$ l o l

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$\color{red}{[}$ l $\color{red}{\rangle}$ o l

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$[$ `l` `o` $\rangle$ `l`

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`l  o [⟩ l`

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```

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```
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```

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    ```
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    ```

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$\rightarrow$ We need a **different way** to measure complexity

Enumeration Algorithms

Idea: In real life, we do not want to compute all the matches
we just need to be able to enumerate matches quickly

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how to find patterns | Search

Results **1 - 20** of **10,514**

...

View (previous 20 | next 20) (20 | 50 | 100 | 250 | 500)

Enumeration Algorithms

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→ Formalization: enumeration algorithms

Formalizing Enumeration Algorithms

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Text *T*

␣ [a-z0-9.]*@
[a-z0-9.]* ␣

Pattern *P*

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Phase 1:

Preprocessing

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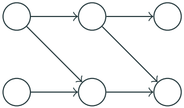
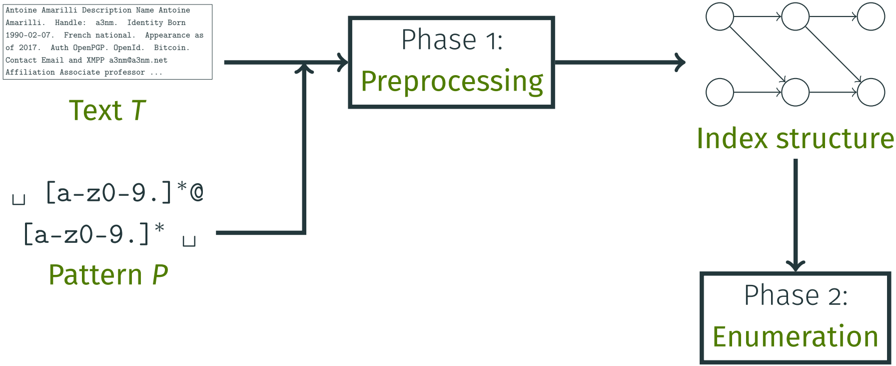


Index structure

Formalizing Enumeration Algorithms



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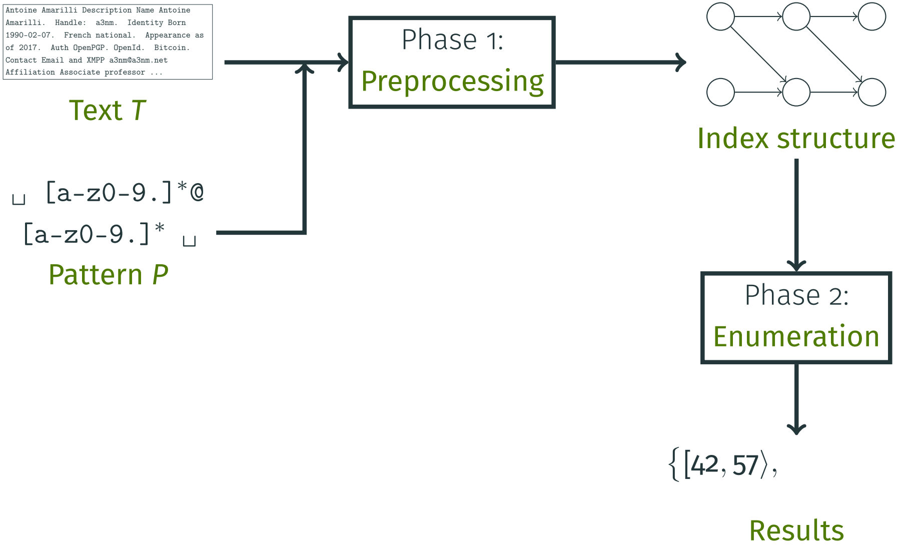

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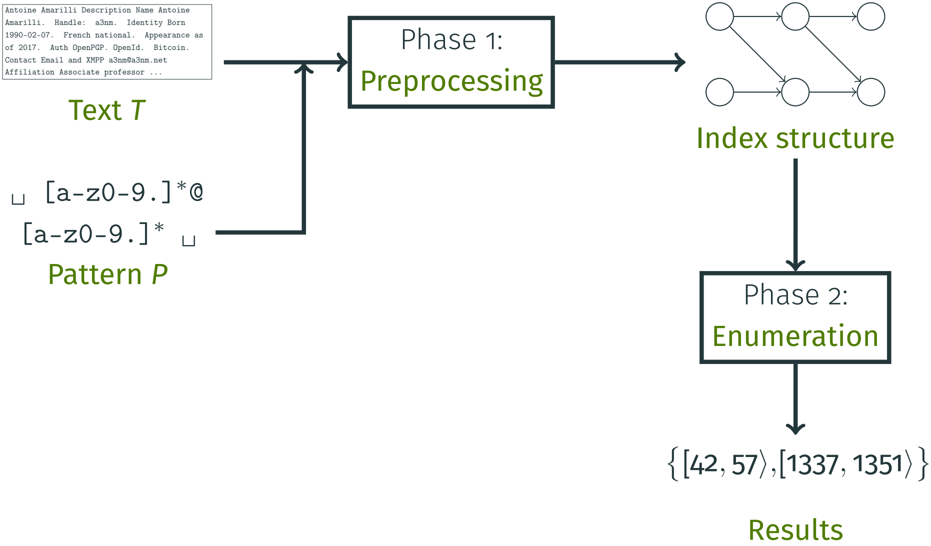

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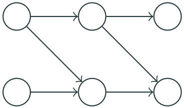

Index structure

Phase 2: **Enumeration**

$\{[42, 57\rangle, [1337, 1351\rangle\}$

Results

Two ways to measure performance:

- **Total time** for phase 1
- **Delay between two results** in phase 2

... as a function of the **text** and **pattern**

Complexity of Enumeration Algorithms

- Recall the inputs to our problem:
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→ Can we do **better**?

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Theorem [Florenzano et al., 2018]

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Theorem

We can enumerate all matches of a pattern P on a text T with:

- *Preprocessing in $O(|T| \times Poly(P))$*
- *Delay polynomial in P and independent from T*

Automaton Formalism

- We use automata that read letters and capture variables

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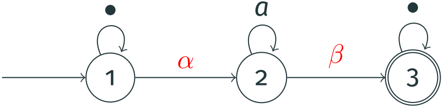

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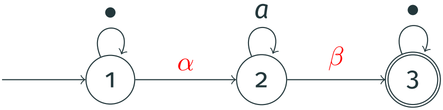

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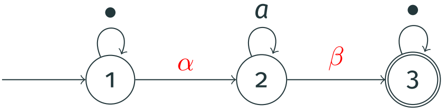

- Semantics of the automaton A:
  - **Reads** letters from the text
  - **Guesses** variables at positions in the text
  - $\rightarrow$ **Output:** tuples $\langle \alpha : i, \beta : j \rangle$ such that A has an accepting run reading α at position i and β at j

Automaton Formalism

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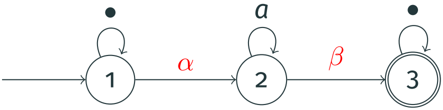
  - $\rightarrow$ Example: $P := \bullet^* \ \alpha \ a^* \ \beta \ \bullet^*$



- Semantics of the automaton A:
  - Reads letters from the text
  - Guesses variables at positions in the text
  - $\rightarrow$ Output: tuples $\langle \alpha : i, \beta : j \rangle$ such that A has an accepting run reading α at position i and β at j
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Automaton Formalism

- We use automata that read letters and capture variables
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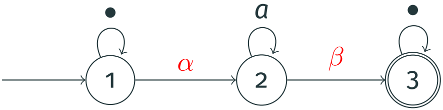

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- Assumption: There is no run for which A reads the same capture variable twice at the same position
- Challenge: Because of nondeterminism we can have many different runs of A producing the same tuple!

Proof Idea: Product DAG

Compute a product DAG of the text T and of the automaton A

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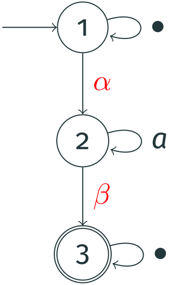

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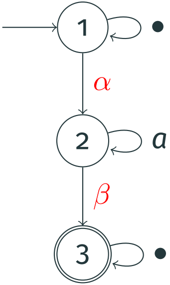
a	a	a	b	a



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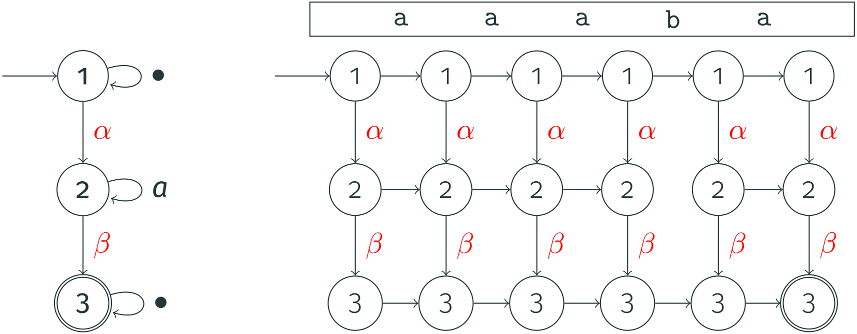

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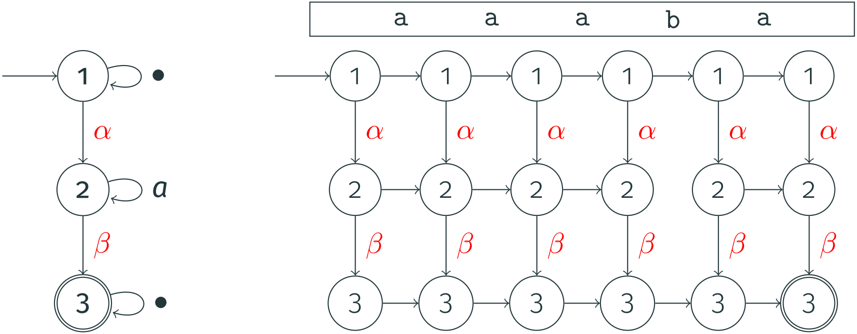
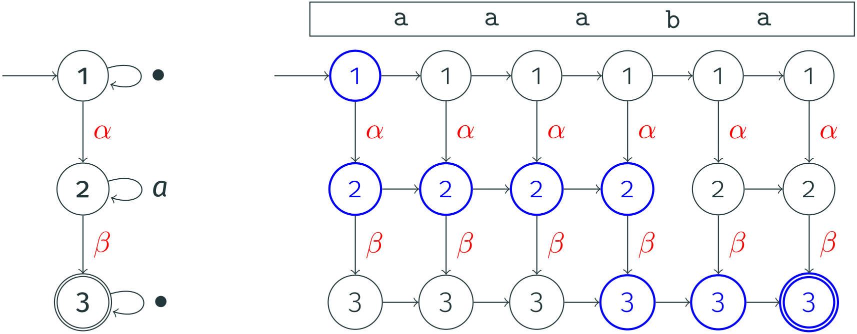


→ Each **path** in the **product DAG** corresponds to a **match**

Proof Idea: Product DAG

Compute a **product DAG** of the text T and of the automaton A

Example: Text $T :=$ $\boxed{\texttt{aaaba}}$ and $P := \bullet^* \ \alpha \ a^* \ \beta \ \bullet^*$, match $\langle \alpha : 0, \beta : 3 \rangle$



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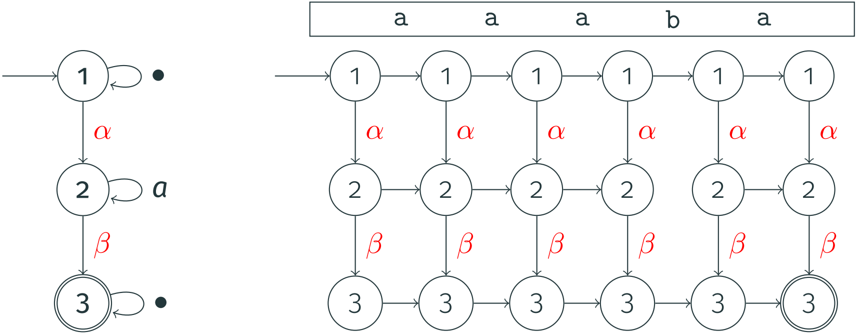
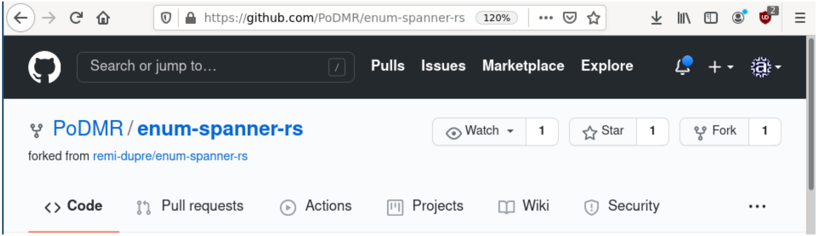


$\rightarrow$ Each **path** in the **product DAG** corresponds to a **match**

$\rightarrow$ **Challenge:** Enumerate paths but avoid **duplicate matches** and do not **waste time** to ensure constant delay

Implementation and Experiments



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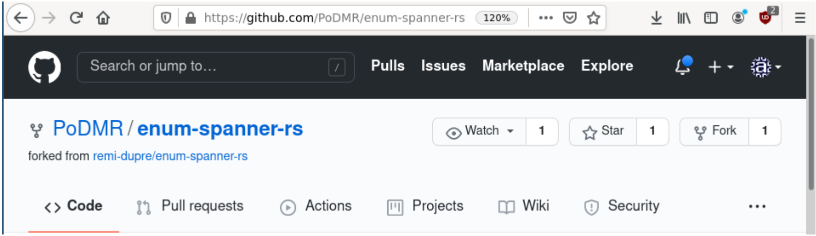

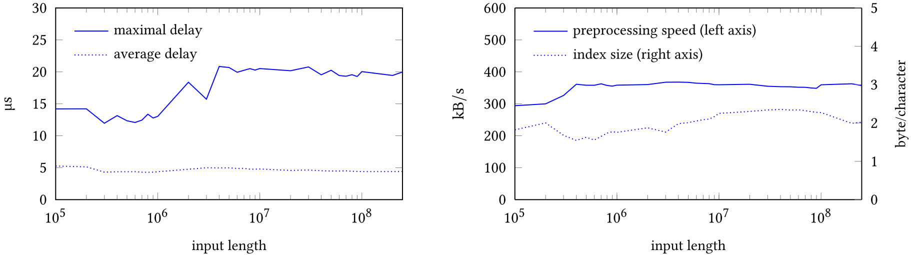

(a) Enumeration delay

(b) Preprocessing speed and index structure size

Fig. 2. Enumerating the query `TTAC.{0,1000}CACC` on inputs of different lengths

Ongoing research and future work

With *P. Bourhis, R. Dupré, M. Niewerth, S. Mengel*:

Efficient implementation of the approach

`https://github.com/PoDMR/enum-spanner-rs`

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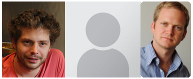

Can we enumerate for **context-free languages**?

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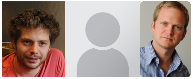

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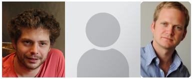

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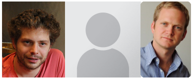

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Thanks for your attention!

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In *ICDT*.

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Florenzano, F., Riveros, C., Ugarte, M., Vansummeren, S., and Vrgoc, D. (2018).
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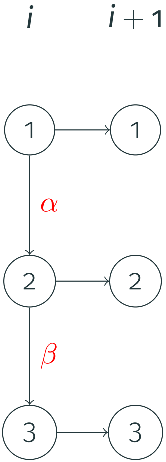
Proof idea: on-the-fly computation to avoid duplicates



- We are at a position i and set of states in blue

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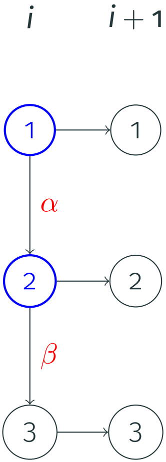

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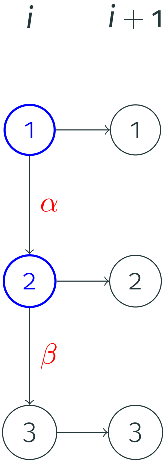

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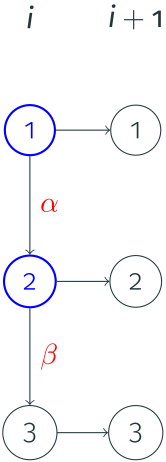

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- Partition tuples based on the set S of variables assigned at the current position
- For each S, consider the set of states where we can be at $i+1$ when reading S at i

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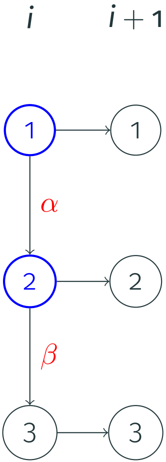

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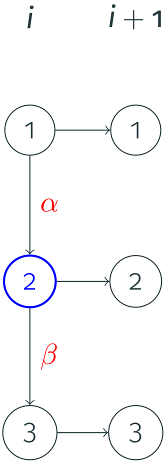

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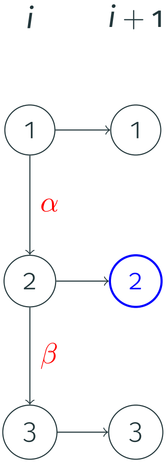

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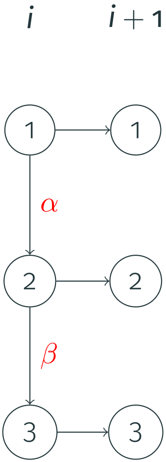

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$\rightarrow$ We must have preprocessed the DAG to make sure that we can always finish the run

Proof idea: jump pointers to save time

- Issue: When we can't assign variables, we do not make progress

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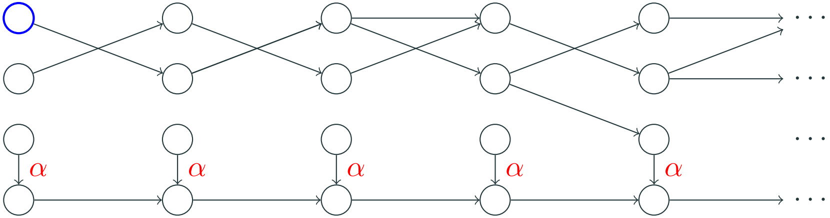

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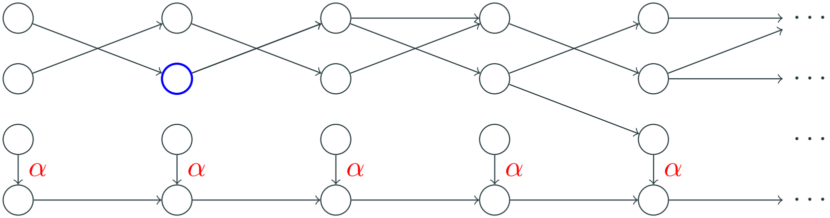

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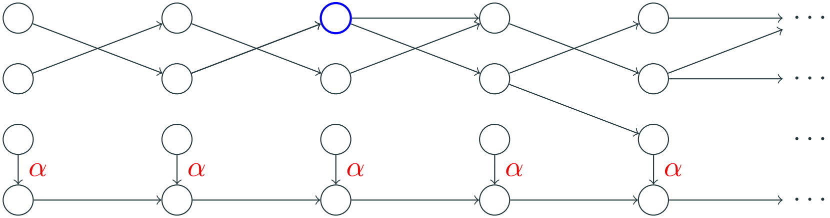

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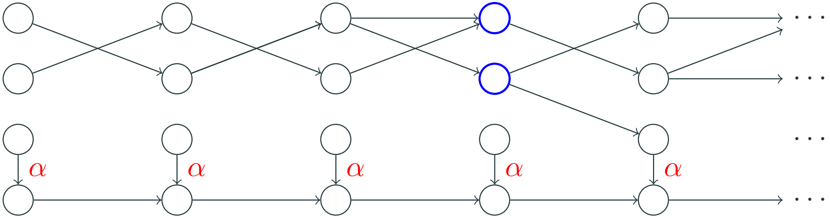

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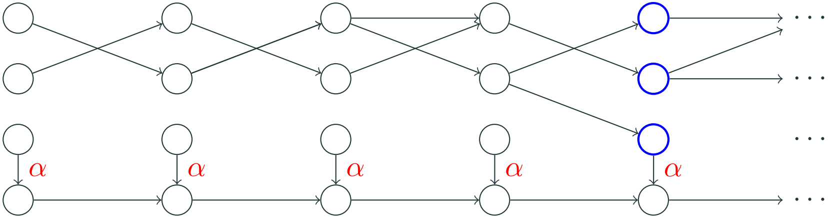

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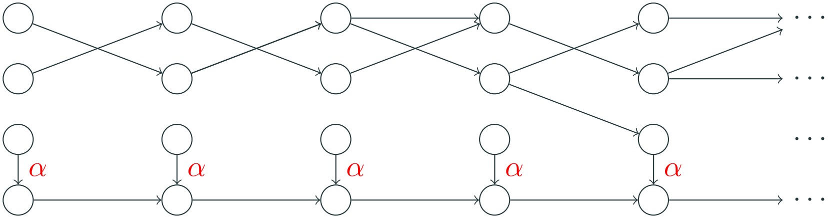

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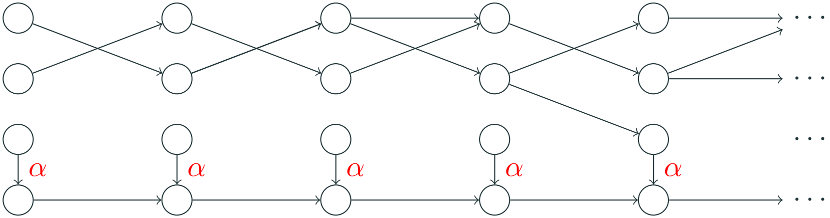

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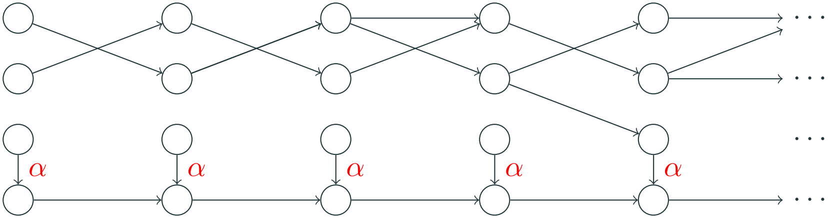

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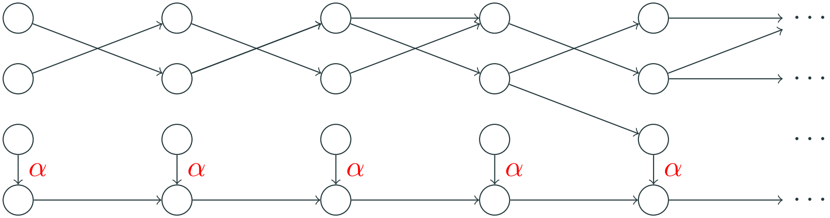

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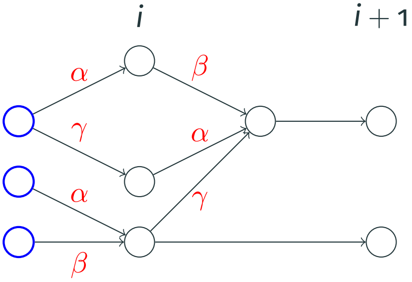
  - $\rightarrow$ Compute at each position i the **transitive closure** to all positions j such that j is the next position of some state at i (there are $\leq |A|$)

Proof idea: flashlight search

- **Issue:** Finding which **variable sets** we can assign at position i?



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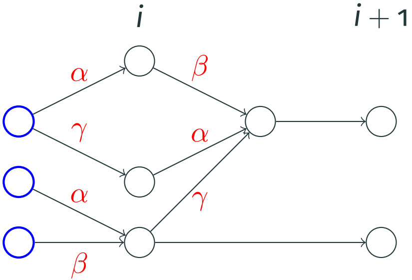

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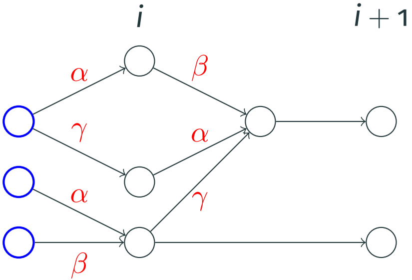

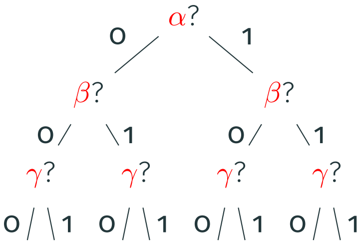

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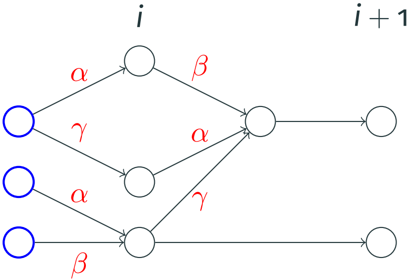

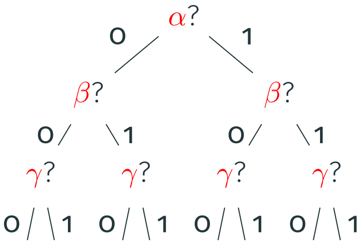

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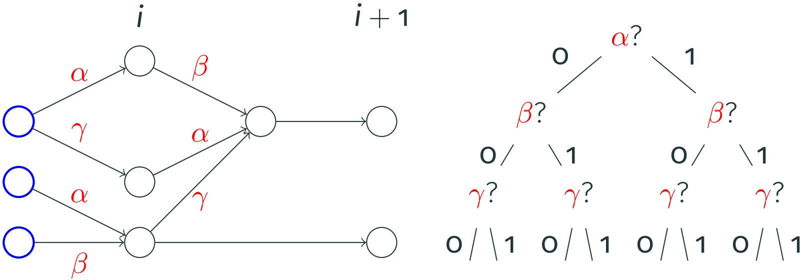

- Idea: Explore a **decision tree** on the variables (built on the fly)
- At each decision tree **node**, find the reachable **states** which have **all required variables** (1) and **no forbidden variables** (0)
  - $\rightarrow$ **Assumption**: we don't see the same variable **twice** on a path